Wave Motion 51 (2014) 52–59

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Special solitonic localized structures for the (3 + 1)-dimensional Burgers equation in water waves

Chao-Qing Dai [a,*], Fang-Bo Yu [b]

[a] School of Sciences, Zhejiang A&F University, Lin'an, Zhejiang 311300, PR China

[b] School of Environmental & Resources, Zhejiang A&F University, Lin'an, Zhejiang 311300, PR China

HIGHLIGHTS

- A modified mapping method for NPDEs.
- Variable separation solution for the 3 + 1D Burgers equation.
- Elastic interactions between rectangle and ring solitons on background.

ARTICLE INFO

Article history:
Received 8 November 2012
Received in revised form 11 June 2013
Accepted 19 June 2013
Available online 27 June 2013

Keywords:
(3 + 1)-dimensional Burgers equation
Modified mapping method
Variable separation solution
Interaction between special solitons

ABSTRACT

With the help of a modified mapping method, we re-study the (3+1)-dimensional Burgers equation and derive three families of variable separation solutions. By selecting appropriate functions in the variable separation solution, we discuss interaction behaviors among flat-top rectangle-soliton and ring-soliton, embedded rectangle-soliton and embedded ring-soliton in a periodic wave background. All interaction behaviors among them are completely elastic, and no phase shift appears after the interaction. These results might be helpful to the understanding of the propagation processes for nonlinear water waves in fluid mechanics such as diverse nonequilibrium, nonlinear phenomena in turbulence and inter-face dynamics.

1. Introduction

In the past decades, shallow water waves and a host of long wave phenomena are commonly investigated by various models of nonlinear partial differential equations (PDEs). Many effective methods were established to solve these nonlinear PDEs, such as the inverse scattering method [1,2], Hirota method [3], the similarity transformation method [4] and F-expansion method [5], etc. In linear mathematical physics, the Fourier analysis and the variable separation approach (VSA) are two most universal and powerful tools to investigate linear PDEs. As the counterpart of the Fourier analysis, the celebrated inverse scattering method [1,2] plays an important role in analyzing nonlinear wave dynamics in nonlinear mathematical physics. The VSA [6–14] to solve nonlinear PDEs has also been established. The direct and simple one is the VSA based on the mapping method [8,9], which has different forms including the improved projective approach [10,11], the q-deformed hyperbolic functions method [12] and the projective Riccati equation method [13,14].

Based on various variable separation solutions, abundant localized coherent structures have been investigated. In 1 + 1 dimension [15], several typical soliton structures, including bell-type, kink-type, peaked, compacted, W-shaped type, M-shaped type and loop solitons etc., have been discussed. In 2 + 1 dimension [14], rich novel localized coherent structures, such as dromion, peakon, compacton, foldon, nonpropagating solitons, instanton, ghoston, lump and ring

* Corresponding author. Tel.: +86 0571 63754660; fax: +86 0571 63754660.
E-mail addresses: dcq424@126.com, dcq424@163.com (C.-Q. Dai).

0165-2125/$ – see front matter
http://dx.doi.org/10.1016/j.wavemoti.2013.06.002

solitons, oscillating soliton, fractal and chaotic solitons, etc., have been investigated. In 3+1 dimension, some exotic localized coherent structures, e.g. paraboloid-type soliton and dipole-type dromion [16], doubly periodic wave solutions [17], coaxial cylindraceous solitons [18], embedded soliton [19] and plateau like soliton on the kink background [20] and so on, have also been extensively studied. Compared with soliton structures of 1 + 1 and 2 + 1 dimensions, those of 3 + 1 dimension are more important since the real space is three dimension. However, 3+1 dimensional structures were relatively less reported than 1+1 and 2+1 dimensional structures in the previous literature, and thus there are still many interesting issues worth studying further.

Now there are several interesting issues that arise, i.e. whether other mapping equation can be used to obtain variable separation solutions of some (3+1)-dimensional nonlinear physics systems. Can we discuss some new (3+1)-dimensional dynamical behaviors based on these variable separation solutions? In order to answer these issues, we study the following well-known (3 + 1)-dimensional Burgers equation:

$$u_t = 2uu_y + 2vu_x + 2wu_z + u_{xx} + u_{yy} + u_{zz}, \tag{1}$$

$$u_x = v_y, \qquad u_z = w_y, \tag{2}$$

as a concrete example. This equation describes the propagation processes for nonlinear waves in fluid mechanics such as diverse nonequilibrium, nonlinear phenomena in turbulence and inter-face dynamics. If u is z-independent (or $z = x$; $w = u$), (2) will be degenerated to the known (2+1)-dimensional Burgers equation. Furthermore, if u is both z-independent and y-independent (or $y = z = x$; $v = w = u$), Eq. (2) is just the well-known (1 + 1)-dimensional Burgers equation which is widely applied in many scientific fields. An alternative potential form of Eq. (2) is obtained from the invertible deformation of the heat conduction equation [21]. In Refs. [16,19,22], authors extended the VSA to this system.

2. The modified mapping method

For a given nonlinear PDE with independent variables $x = (x_0 = t, x_1, x_2, x_3, \ldots, x_m)$, and dependent variable u,

$$L(u, u_t, u_{x_i}, u_{x_ix_j}, \ldots) = 0, \tag{3}$$

where L is in general a polynomial function of its argument, and the subscripts denote the partial derivatives.

The basic idea of the mapping method is to seek for its ansätz

$$u = a_0(x) + \sum_{i=1}^{n} \left\{ a_i(x)\phi^i[q(x)] + \frac{b_i(x)}{\phi^i[q(x)]} + c_i(x)\phi^{i-1}[q(x)]\sqrt{\phi'[q(x)]} \right\}, \tag{4}$$

where a_i, b_i, c_i are arbitrary functions of $\{x\}$ to be determined and n is fixed by balancing the linear term of the highest order with the nonlinear term in Eq. (3), ϕ satisfies a mapping equation [7–11] and q is an arbitrary function of $\{x\}$. Here the superscript i indicates the power of ϕ and the prime denotes differentiation with respect to q.

Note that many mapping equations for ϕ have been used, such as the Riccati equation $\phi' = l_0 + \phi^2$ (l_0 is a constant) [7–9], $\phi' = \sigma\phi + \phi^2$ (σ is a constant) [10,11] and $\phi' = l_1 + l_2\phi^2$ (l_1 and l_2 are two constants) [23]. Here we seek for its solution of the given nonlinear PDE (3) with a mapping equation [24]

$$\phi' = (A\phi - C)(B\phi - D), \tag{5}$$

which is known to possess the general solution

$$\phi = \frac{D\exp[(BC - AD)q] - C\exp[C_1(AD - BC)]}{B\exp[(BC - AD)q] - A\exp[C_1(AD - BC)]}. \tag{6}$$

Here C_1 is an integration constant, further, A, B, C and D are arbitrary constants.

To determined u explicitly, we take the following three steps:

Step 1: Determine n by balancing the highest nonlinear terms and the highest-order partial differential terms in the given nonlinear PDE (3).

Step 2: Substituting Eq. (4) along with Eq. (5) into Eq. (3) yields a set of polynomials for $\phi^i\sqrt{\phi'}$. Eliminating all the coefficients of the powers of $\phi^i\sqrt{\phi'}$ yields a series of partial differential equations, from which the parameters a_i, b_i, c_i and q are explicitly determined.

Step 3: Substituting a_i, b_i, c_i, q and Eq. (6) into Eq. (4), one can obtain possible solutions of Eq. (3).

3. Variable separation solutions for the (3 + 1)-dimensional Burgers equation

Along with the modified mapping method in Section 2, by balancing the highest-order derivative terms with the nonlinear terms in Eq. (2), we suppose that it has the following formal solutions: