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Surface and interfacial waves in anisotropic elastic quasicrystals

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HIGHLIGHTS

- Stroh formalism for two-dimensional subsonic motion of anisotropic quasicrystals.
- At most three subsonic surface wave speeds in anisotropic quasicrystals.
- At most one surface wave in half-space for a traction-free and insulating surface.
- At most two subsonic surface wave speeds for a traction-free and conducting surface

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ABSTRACT

We present the Stroh formalism for two-dimensional subsonic steady-state motion of anisotropic quasicrystals. Using this new formalism and a series of identities and properties which follow, we investigate subsonic surface and interfacial waves in anisotropic quasicrystals. Our results suggest that there exist at most three subsonic surface wave speeds. This interesting observation is quite different from the unique surface wave speed known for anisotropic crystals. The degenerate case of decagonal quasicrystalline materials is discussed in detail.

1. Introduction

The study of surface waves in anisotropic elastic crystals has a long history (see, for example, [1–7]) which includes the derivation of many excellent theoretical results concerning the uniqueness and existence of surface waves in anisotropic crystals. Brief reviews and comments on these results can be found in [5,8]. A counterpart of the surface wave propagating along the surface of a half-space is the so-called edge wave propagating along the edge of a thin plate [9,10]. Interestingly, both surface and edge waves in anisotropic media can be investigated by means of the Stroh method: the surface wave in anisotropic solids can be studied by means of the standard sextic formalism [6,8], whilst the edge wave in Kirchhoff anisotropic plates can be addressed using a newly developed Stroh-like octet formalism [11,10]. The major difference between surface and edge waves lies in the fact that if a subsonic surface wave exists it is unique [3,8] whereas there can exist at most two edge-wave speeds [10].

Quasicrystals, which were first discovered by Shechtman et al. [12] nearly two decades ago, possess a type of ordered structure characterized by crystallographically disallowed long-range orientational symmetry and by long-range quasi-

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periodic translational order. These materials exhibit very unusual features including a very high electrical and thermal resistivity, increased hardness and brittle behavior at room temperature which make them extremely attractive in a wide range of existing and emerging applications. In this paper, we begin (in Section 2) by extending the Stroh formalism for two-dimensional elastostatics of anisotropic quasicrystals [13,14] to subsonic steady-state motion of anisotropic quasicrystals. Some identities and properties necessary for our subsequent theoretical development are derived in Section 3. In Section 4 we investigate subsonic surface waves in anisotropic quasicrystals and arrive at the interesting result that there exist at most three subsonic surface wave speeds. Interfacial waves in anisotropic quasicrystalline bimaterials are discussed in Section 5. Finally the degenerate case of decagonal quasicrystalline materials is discussed in detail in Section 6.

2. Stroh formalism for steady-state motion

2.1. Reduction to elastostatic equations

In a fixed rectangular coordinate system x_i $(i = 1, 2, 3)$, let u_i, w_i be the phonon and phason displacements and σ_{ij} $(\sigma_{ij} = \sigma_{ji})$, H_{ij} $(H_{ij} \neq H_{ji})$ be the phonon and phason stresses, respectively, in an anisotropic quasicrystalline material. The stress–strain law and the equations of motion are [15]

$$\begin{aligned} &\sigma_{ij} = C_{ijkl}u_{k,l} + R_{ijkl}w_{k,l}, \qquad H_{ij} = R_{klij}u_{k,l} + K_{ijkl}w_{k,l}, \\ &\sigma_{ij,j} = \rho\ddot{u}_i, \qquad H_{ij,j} = 0, \end{aligned} \tag{1}$$

where ρ is the mass density, comma and dot denote differentiation with respect to x_i and time t, respectively, C_{ijkl} are the elastic constants in the phonon field, K_{ijkl}, are the elastic constants in the phason field and R_{ijkl} are the phonon–phason coupling constants. In addition C_{ijkl}, R_{ijkl} and K_{ijkl} possess the following symmetry:

$$C_{ijkl} = C_{jikl} = C_{klij} = C_{ijlk}, \qquad R_{ijkl} = R_{jikl}, \qquad K_{ijkl} = K_{klij}. \tag{2}$$

In writing Eq. (1), we have neglected the dissipation associated with the atomic rearrangements [16]. For a steady state motion in the x_1-direction with a constant speed $v > 0$, the displacements take the following forms

$$u_i = u_i(x_1 - vt, x_2, x_3), \qquad w_i = w_i(x_1 - vt, x_2, x_3), \tag{3}$$

from which one can obtain

$$\dot{u}_i = -vu_{i,1}, \qquad \ddot{u}_i = -v\dot{u}_{i,1}. \tag{4}$$

If we introduce the following notations

$$\begin{aligned} &\hat{\sigma}_{ij} = \sigma_{ij} + \rho v\dot{u}_i\delta_{j1}, \\ &\hat{C}_{ijkl} = C_{ijkl} - \rho v^2\delta_{ik}\delta_{j1}\delta_{l1}, \end{aligned} \tag{5}$$

where δ_{ij} is the Kronecker delta, Eq. (1) can be rewritten as

$$\begin{aligned} &\hat{\sigma}_{ij} = \hat{C}_{ijkl}u_{k,l} + R_{ijkl}w_{k,l}, \qquad H_{ij} = R_{klij}u_{k,l} + K_{ijkl}w_{k,l}, \\ &\hat{\sigma}_{ij,j} = 0, \qquad H_{ij,j} = 0, \end{aligned} \tag{6}$$

which are identical to the stress–strain law and equilibrium equations for elastostatics [14].

2.2. The Stroh formalism

For two-dimensional deformations in which u_i and w_i depend only on $\hat{x}_1 = x_1 - vt$ and x_2, the general solutions can be expressed as

$$\begin{aligned} &\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & w_1 & w_2 & w_3 \end{bmatrix}^T = \mathbf{Af}(z) + \bar{\mathbf{A}}\overline{\mathbf{f}(z)}, \\ &\mathbf{\Phi} = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \Psi_1 & \Psi_2 & \Psi_3 \end{bmatrix}^T = \mathbf{Bf}(z) + \bar{\mathbf{B}}\overline{\mathbf{f}(z)}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} &\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 & \mathbf{a}_5 & \mathbf{a}_6 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 & \mathbf{b}_4 & \mathbf{b}_5 & \mathbf{b}_6 \end{bmatrix}, \\ &\mathbf{f}(z) = \begin{bmatrix} f_1(z_1) & f_2(z_2) & f_3(z_3) & f_4(z_4) & f_5(z_5) & f_6(z_6) \end{bmatrix}^T, \\ &z_i = \hat{x}_1 + p_i x_2, \qquad \operatorname{Im}\{p_i\} > 0, \quad (i = 1\text{–}6), \end{aligned} \tag{8}$$