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Plane waves in a fractional order micropolar magneto-thermoelastic half-space

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HIGHLIGHTS

- We model a fractional order generalized micropolar magneto-thermoelastic medium.
- The potential function approach along with Laplace and Fourier transforms is employed.
- Different field variables are obtained using a numerical inversion technique.
- Significant effects of fractional parameter, magnetic field and micropolarity are observed.
- Numerical results predict finite speed of propagation for thermoelastic waves.

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ABSTRACT

This paper deals with the problem of magneto-thermoelastic interactions in an unbounded, perfectly conducting half-space whose surface suffers a time harmonic thermal source in the context of micropolar generalized thermoelasticity with fractional heat transfer allowing the second sound effects. The medium is assumed to be unstrained and unstressed initially and has uniform temperature. The Laplace–Fourier double transform technique has been used to solve the resulting non-dimensional coupled field equations. Expressions for displacements, stresses and temperature in the physical domain are obtained using a numerical inversion technique. The effects of fractional parameter, magnetic field and micropolarity on the physical fields are noticed and depicted graphically. For a particular model, these fields are found to be significantly affected by the above mentioned parameters. Some particular cases of interest have been deduced from the present problem. Numerical results predict finite speed of propagation for thermoelastic waves.

1. Introduction

Eringen's micropolar theory of elasticity [1] is now well known and does not need much introduction. A historical development of the theory of micropolar elasticity is given in a monograph by Eringen [2]. In this theory, a load across a surface element is transmitted not only by a force stress vector but also by a couple stress vector and the motion is characterized by six degrees of freedom (three of translation and three of microrotation). Micropolar elastic solids can be thought of as being composed of dumb-bell type molecules and these molecules in a volume element can undergo rotation about their centre of mass along with the linear displacement. This theory is expected to find applications in the treatment of mechanics of granular materials, composite fibrous materials and particularly microcracks and microfractures.

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Notations

σ_{ij}	Components of force stress tensor
m_{ij}	Components of couple stress tensor
$\vec{\phi}$	Microrotation vector
β_1	$= (3\lambda + 2\mu + k)\,\alpha_t$
θ	$= T - T_0$
α, β, γ, k	Micropolar material constants
$\lambda,\ \mu$	Lame's constants
α_t	Coefficient of linear thermal expansion
j	Microinertia
T	Absolute temperature
T_0	Temperature of the medium in its natural state assumed to be $\lvert\theta/T_0\rvert \ll 1$
u_i	Components of the displacement vector
ρ	Density of the medium
e_{ij}	Components of the strain tensor
e	Cubical dilatation
c_E	Specific heat at constant strain
K^*	Thermal conductivity
τ_0	Thermal relaxation time
$\vec{F}$	Lorentz force
μ_0	Magnetic permeability
ε_0	Electric permittivity
Γ	Gamma function
m	Fractional order parameter such that $0 < m \le 1$

The dynamical interaction between thermal and mechanical fields in solids has great applications in aeronautics, nuclear reactors and high energy particle accelerators. Keeping the above applications in view, the micropolar theory was extended to include thermal effects by Nowacki [3–5] and Eringen [6]. Tauchert et al. [7] also derived the basic equations of the linear theory of micropolar thermoelasticity. Among the contributions to the subject of micropolar thermoelasticity are the works of Shanker and Dhaliwal [8], who have solved several dynamic thermoelastic problems in micropolar theory. Chirita [9] proved the existence and uniqueness theorems for the equations of linear coupled thermoelasticity with microstructures.

In recent years increasing attention has been directed towards the generalized theory of thermoelasticity, which is found to predict more realistic results than the coupled or uncoupled theories of thermoelasticity, especially when short time effects or step temperature gradients are considered. A generalized theory of linear micropolar thermoelasticity that admits the possibility of 'second sound' effects was established by Boschi and Iesan [10] using the Green and Lindsay theory [11]. The basic equations for this theory are derived using invariance conditions under superposed rigid body motions. Dost and Tabarrok [12] also proposed the generalized micropolar thermoelasticity theory based on Green–Lindsay model. As an illustrative example, the propagation of acceleration waves is investigated and the acoustic tensor, which determines the speed of propagation, is obtained. A theory of micropolar thermoelasticity without energy dissipation that permits propagation of thermal waves at a finite speed was suggested by Ciarletta [13]. In this theory a Galerkin type solution is used to determine the effect of a concentrated heat source in an unbounded domain. Sherief et al. [14] presented the generalized equations for the linear theory of micropolar thermoelasticity based on Lord–Shulman model [15]. A uniqueness theorem is also provided in the same article. As an illustration of the obtained equation, they have solved a half-space problem using Laplace and Hankel transforms whose boundary is rigidly fixed and subjected to an axi-symmetric thermal shock.

Several researchers have shown much interest to discuss the phenomenon of wave propagation in micropolar generalized thermoelastic medium due to its multifarious applications. It finds applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessels, aerospace, chemical pipes and metallurgy. Steady-state response of micropolar thermoelastic half-space to moving load in the context of generalized thermoelasticity theories has been analyzed by Kumar and Deswal [16]. Kumar and Deswal [17] investigated the disturbances due to mechanical and thermal sources in a micropolar generalized thermoelastic half-space in a unified formulation of Lord–Shulman and Green–Lindsay models using Laplace and Fourier transforms. Othman and Singh [18] contributed their research efforts to examine the effect of rotation on generalized micropolar thermoelasticity using normal mode analysis under various theories. Kumar and Deswal [19] also participated in the development of micropolar generalized thermoelasticity by solving an axi-symmetric half-space problem. Ciarletta et al. [20] considered the problem of plane waves and vibrations within the framework of micropolar thermoelasticity with voids and reported some basic properties of wave numbers of the longitudinal and transverse plane harmonic waves. Recently, Shaw and Mukhopadhyay [21] studied the phenomenon of thermoelastic wave propagation in isotropic micropolar plate with insulated as well as isothermal boundary conditions.