Wave Motion 51 (2014) 125–145

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Reciprocity identities for quasi-static piezoelectric transducer models: Application to cavity identification using iterated excitations and a topological sensitivity approach

Cédric Bellis, Sébastien Imperiale [*]

Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY 10027, USA

HIGHLIGHTS

- Derivation of a time-domain reciprocity identity for a piezoelectric sensor model.
- Definition of an iterative procedure to construct optimal electric excitations.
- Adapted topological sensitivity approach to detect scattering obstacles.
- 2D finite-element computations to assess the performances of the approach.

ARTICLE INFO

Article history:
Received 24 December 2012
Received in revised form 27 April 2013
Accepted 21 May 2013
Available online 8 July 2013

Keywords:
Piezoelectric transducers
Reciprocity identity
Iterated time reversal
Adjoint method
Topological sensitivity

ABSTRACT

The focus of this article is the transient wave-based detection and identification of defects embedded in isotropic elastic solids using piezoelectric transducers. This work addresses this problem within a comprehensive framework encompassing description of elastic wave propagation within the probed media as well as consideration of the coupling phenomena induced by the transducers. A fundamental reciprocity identity associated with a quasi-static piezoelectric model is derived to lay the foundations of ensuing developments and approach of this inverse scattering problem. Modeling of piezoelectric transducers is discussed and application of the proven reciprocity theorem enables the proposition of an iterative construction procedure of electric inputs generating waves expected to focus on the sought defects. The characteristic features of the inverse problem considered, which uses piezoelectric sensor-based measurements, are also discussed. Next, the identification problem is investigated by way of an adjoint field-based topological sensitivity approach that permits the construction of a defect indicator function based on the derived reciprocity identity. For simplicity of exposition, the studied configurations involve defects in the form of traction-free cavities. Finally, a set of 2D numerical examples based on the spectral finite-elements method is presented to assess the performances of the proposed approach in identifying embedded defects from electric measurements.

1. Introduction

Transient elastic or ultrasonic waves are preferred phenomena to probe elastic solids in applications such as non-destructive material testing [1,2]. Possible embedded and unknown defects, i.e. localized heterogeneities or geometrical features such as cavities or cracks, are illuminated by waves propagating in the solid body considered while measurements

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[*] Corresponding author. Tel.: +331 72 92 59 00.
E-mail address: sebastien.imperiale@inria.fr (S. Imperiale).

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http://dx.doi.org/10.1016/j.wavemoti.2013.05.007

of their scattered counterparts are collected on a subset of the external surface. Based on such boundary data, a wide range of algorithms aiming at detecting and reconstructing such scattering obstacles have been developed over the last few decades. For instance, optimization-based approaches are generally concerned with the characterization of a finite number of parameters that are quantified by minimization of cost-functionals exploiting the available data; see e.g. [3,4]. Alternatively, computationally-efficient techniques centered on the construction and the non-iterative computation of indicator functions of the sought defects have been more recently developed [5–7]. These approaches are commonly referred to as *qualitative methods* [8].

There exists a variety of devices providing full-waveform or partial measurements of boundary displacement fields based on, e.g., mechanical vibrations or electroactive actuators. In this study, it is assumed that the elastic solids of interest are probed using ultrasonic piezoelectric transducers. Such transducers are made of piezoelectric materials which have the property to convert mechanical energy into electric energy, and reciprocally [9]. Here, the piezoelectric phenomena are investigated within the framework of the quasi-static piezoelectric model [10] which features the elastodynamics equation coupled to Maxwell's equations reduced to a scalar electric potential constituting the quantity which can be controlled and recorded during an experiment. In the configuration considered, such a transducer is placed in contact with the investigated elastic medium and it is used both as the source of the illuminating elastic waves as well as, as the receiver of the associated echoes [11,12]. In emission-mode, customized electric potentials are applied on the active elements of the transducer, which generates a wave propagating in the underlying solid. Alternatively, during the reception regime, or sensor-mode, electric currents associated with the displacement field generated by the mechanical waves impinging the sensor are recorded.

In the present study, the defect identification problem is addressed in a comprehensive framework in dimension two or three, which encompasses description of elastic wave propagation within the probed medium as well as of companion transient piezoelectric phenomena occurring within the transducer. This approach is based on the recent progress concerning the mathematical treatment of piezoelectric transducer models in association with the development of a performant and highly accurate simulation tool [10]. Therefore, the fully-coupled problem arising due to the presence of the piezoelectric transducer is taken into account. Some specific issues arise in this context: first, the presence of the piezoelectric transducer induces an electro-mechanical coupling impacting the elastic field within the probed medium and thus its observation in comparison with a configuration featuring purely elastic boundary excitation and measurements. Moreover, only time-dependent and discrete scalar measurements of the electric field, rather than full-waveform data, are accessible. Finally, in their most general form, the electric measurements are associated with an integral operator, in time and space, acting on the elastodynamic state associated with the echoes recorded at the sensor's interface with the probed medium. In other words, the mapping between boundary elastic field and measured electric potentials lacks of injectivity and the inverse problem considered is severely ill-posed.

The intended contribution of this work is threefold.

(i) Derivation of a time-domain *reciprocity identity* associated with the quasi-static piezoelectric model considered, a terminology which originates from the Betti reciprocity theorem [13]. Improving simplified models such as [14,15] while particularizing the studies [16,17], this identity, which can be seen as stemming from a weak formulation of the problem or the virtual work principle, corresponds to a cross-relation between Neumann and Dirichlet boundary data associated with two solution states satisfying the same piezoelectric field equations over a given geometrical domain. Reciprocity-based methods are classical techniques in the field of non-destructive testing [18], in particular those based on the so-called reciprocity gap concept [19,20]. Therefore, the derived identity constitutes a key result for the proposed approach of the inverse problem considered and a thread of the ensuing developments.

(ii) Application of the reciprocity theorem to a specific model of piezoelectric transducer, i.e. introducing the set of coupled elastic–electric boundary conditions involved in the emission and reception regimes. This additional result provides the framework for the proposition of an iterative procedure aiming at constructing optimal electric excitations generating elastic waves achieving selective focusing on the unknown defects.

(iii) Construction of an indicator function of the sought scattering obstacles which are considered in the form of traction-free cavities for simplicity of exposition. To that purpose, the chosen approach is based on the concept of topological sensitivity which revolves around the quantification of the perturbation of a given cost functional induced by an infinitesimal defect. This method that has been developed and applied in a variety of configurations [5,21,22,7] is extended to the present context of piezoelectric sensor-based measurements by taking advantages of an adjoint-field formulation [23] which requires application of the previously derived reciprocity identities.

This article is organized as follows. The elastic–electric coupled equations describing the behavior of a piezoelectric solid are presented in Section 2, within the framework of a quasi-static approximation of Maxwell's equations. The fundamental result proven in Section 3 is a theorem establishing a reciprocity identity associated with the piezoelectric model considered in a generic geometrical configuration. Section 4 is concerned with the description of a specific model of piezoelectric transducer, application of the derived reciprocity identity and construction of optimally-focusing excitations. Next, the inverse problem considered, i.e. detection and identification of embedded cavities, is stated in Section 5 and the issues arising when dealing with piezoelectric measurements are discussed. Construction of the indicator function along the lines of the topological sensitivity approach is presented Section 6. Finally, numerical results associated with 2D configurations and based on the spectral finite-elements method are shown and discussed in Section 7.