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On the reflection of waves in half-spaces of microstructured materials governed by dipolar gradient elasticity

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ABSTRACT

The present work studies the propagation and reflection of plane waves in a body having the form of a half-space. It is assumed that the mechanical response of this body is governed by dipolar gradient elasticity. Our aim is to investigate the effect of boundaries on the elastic wave motion in a medium with microstructure and, thus, to determine possible deviations from the predictions of classical linear elastodynamics. The use of the theory of gradient elasticity is intended to model the response of materials with microstructure and incorporate size effects into stress analysis in a manner that the classical theory cannot afford. Here, a simple but yet rigorous version of the Toupin–Mindlin generalized continuum theory is employed that also includes micro-inertial effects. Our results show significant departure from those of classical elastodynamic theory. Indeed, it is observed that an incident dilatational or distortional wave at the traction-free plane boundary gives rise to four reflected waves, instead of the usual two waves predicted by the classical theory. It is shown that the amplitudes, the angles of reflection, and the phase shift of the reflected waves depend significantly upon the material microstructure. This dependence becomes more pronounced at shorter wavelengths.

1. Introduction

Classical continuum theories are inherently size independent, and they are therefore unable to capture the scale effects observed experimentally in problems where the specimen size or the wavelength of the disturbance are comparable to the lengths of the material microstructure. In contrast, *generalized continuum theories* enrich the classical continuum with additional material lengths (characteristic lengths) in order to describe the scale effects resulting from the underlying microstructure. Notable examples, where interesting scale effects have been reported in the context of generalized continua, include torsion and bending of thin wires (Fleck et al. [1], Stolken and Evans [2]), buckling of elastic fibers in composites (Fleck and Shu [3]), micro-indentation experiments where the measured indentation hardness increases as the width of the indent decreases (Poole et al. [4], Huang et al. [5]), and fracture of cellular materials (Chen et al. [6]). In addition, size effects have also been observed in the propagation of elastic waves with short wavelengths in layered and fiber-reinforced media (Achenbach et al. [7], Sun et al. [8]), thin films (Huang and Sun [9]), long bones (Lakes [10], Vavva et al. [11], Papacharalampopoulos et al. [12]), and also in the dynamic analysis of simple structural components (Tsepoura et al. [13]). Furthermore, in wave propagation dealing with electronic-device applications, wave frequencies of gigahertz order are often used, and therefore wavelengths of micron order appear. In such situations, *dispersion phenomena* at high frequencies can be explained on the basis of generalized continuum theories (Georgiadis and Velgaki [14], Georgiadis et al. [15], Papargyri-Beskou et al. [16]). A comprehensive review article on generalized continuum theories has been written recently by Maugin [17].

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One of the most effective generalized continuum theories proved to be in recent years the dipolar gradient theory introduced by Toupin [18] and Mindlin [19]. The general framework appears also under the name 'strain gradient theory' or 'grade-two theory'. According to this, each material particle has three degrees of freedom (the displacement components—just as in the classical theory), and the micro-density does not differ from the macro-density. Also, first-order gradient terms of strain and velocity, in addition to the classical (i.e. zero-order gradient) terms, are included in the strain and the kinetic energy densities, respectively (see Section 2). We notice that the gradient of strain comprises both rotation and stretch gradients. Therefore, this theory is more general than the Cosserat theory with constrained rotations (we call this theory throughout this paper 'standard couple-stress theory') assuming a strain energy density that depends upon the strain and the gradient of rotation (torsion–flexure tensor) only. Also, the dipolar gradient theory is different from the Cosserat (or micropolar) theory that takes material particles with six independent degrees of freedom (three displacement components and three rotation components, the latter involving rotation of a micro-medium with respect to its surrounding medium). Finally, we note that, due to the dependence of the strain energy density on gradients of the strain in the Toupin–Mindlin theory, the new material constants imply the presence of characteristic lengths in the material behavior. These lengths can be related with the size of microstructure (see e.g. Fleck and Hutchinson [20]). Typical cases of continua amenable to such an analysis are material structures like those, for example, of cellular materials, granular materials, foams, and polymers.

The Toupin–Mindlin theory had already some successful applications mainly on stress-concentration problems during the 1960s and 1970s (see e.g. Cook and Weitsman [21], Hermann and Achenbach [22], Eshel and Rosenfeld [23]). More recently, this approach and related gradient theories have been employed to analyze various problems involving, among other areas, fracture (see e.g. Huang et al. [24], Shi et al. [25], Georgiadis [26], Grentzelou and Georgiadis [27], Markolefas et al. [28], Gourgiotis and Georgiadis [29], Aravas and Giannakopoulos [30], Sciarra and Vidoli [31]), plasticity (see e.g. Fleck and Hutchinson [20], Vardoulakis and Sulem [32], Gao et al. [33], Huang et al. [34]), stress concentration due to discrete loadings (see e.g. Lazar and Maugin [35], Georgiadis and Anagnostou [36]), and wave propagation (see e.g. Maugin and Miled [37], Vardoulakis and Georgiadis [38], Georgiadis et al. [15,39], Engelbrecht et al. [40], Charalambopoulos and Gergidis [41], Vavva et al. [11], Polyzos and Fotiadis [42]). More specifically, recent work by Georgiadis and co-workers [14,15,38,39] on wave propagation problems showed that the Toupin–Mindlin theory predicts types of elastic waves that are not predicted by the classical theory (SH and torsional surface waves in homogeneous materials), and also predicts dispersion of high-frequency Rayleigh waves (the classical theory fails to predict dispersion of these waves at any frequency). Notice that these phenomena are observed in experiments and also predicted by atomic-lattice analyses (see e.g. Maugin [43]).

In the present work, a simple version of the Toupin–Mindlin dipolar gradient elasticity (Georgiadis [15], Lazar and Maugin [35]) is employed to deal with the propagation and reflection of plane waves in a body having the form of a half-space in conditions of plane strain. Our goal is to investigate the effect of boundaries on the elastic wave motion in a medium with microstructure and, thus, to determine possible deviations from the predictions of classical linear elastodynamics. We note that although the complexity of the theory has been kept at a minimum, by retaining a restricted number of material parameters (i.e. the Lamé constants (λ, μ) and the gradient coefficient c), a *micro-inertia* term was included because previous experience with gradient analyses of surface waves showed that this term is indeed important at high frequencies [14,15,38,39]. In fact, including it in the present problem gives dispersion curves that mostly resemble those obtained by atomic-lattice considerations. This micro-inertia term is included in the formulation by considering the appropriate expression for the kinetic energy density of the material particle. Consequently, this leads to an explicit appearance of the intrinsic material length $2h$, which, in turn, can be associated with the material microstructure. Recently, Polyzos and Fotiadis [42], using a simple one-dimensional lattice model of one-neighbor interaction, reproduced the field equations of Toupin–Mindlin theory and correlated the internal length parameters with the actual microstructure of the material.

Regarding now results related to our problem, we note that Graff and Pao [44] investigated the propagation and reflection of plane elastic waves in a half-space governed by the standard couple-stress theory (Cosserat theory with constrained rotations) without micro-inertia. It was found that mode conversion is more complicated in this case due to the existence of three types of waves, instead of the usual two types predicted by the classical theory. In fact, it was shown that two of the reflected waves propagate into the medium, while the additional (third) wave is of shear character and decays rapidly from the free surface. The problem of reflection of plane elastic waves in a half-space was also studied by Parfitt and Eringen [45] in the context of micropolar (Cosserat) theory. Their results showed that, in general, an incident wave gives rise to three reflected waves, all propagating into the medium. In this case, the reflected wave system consists of a wave of dilatational type and of two different pairs of waves of coupled shear type. This coupling arises due to the existence of independent (micro) rotations in the micropolar theory.

It should be noted, however, that the theories of standard couple-stress and micropolar elasticity are different from the theory of dipolar gradient elasticity (employed in the present study) and, in general, give results that do *not* share the same general features of solution behavior (see e.g. [27,29,46]). Moreover, we remark that none of the above studies includes the micro-inertia term, which, as we shall have occasion to see later, plays an important role in the reflection of plane waves.

The contents of our paper are as follows. In Section 2, we summarize the basic equations of the Toupin–Mindlin dipolar gradient elasticity and examine the effects of strain gradients in the propagation of plane waves in an infinite medium. It is worth noting that, unlike the case of classical theory, in gradient theory both dilatational and distortional waves become dispersive. The pertinent dispersion relations are examined in detail. Next, in Section 3, we investigate the propagation of plane waves under plane-strain conditions. A general solution is derived with the aid of Lamé displacement potentials.