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Nonlinear dynamics and chaos of magnetized resonant surface waves in a rectangular container

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ABSTRACT

In this paper the nonlinear evolution of the three-dimensional instability of resonated standing surface waves along the interface of a weakly viscous, incompressible magnetic liquid within a rectangular basin is investigated. A combination of the Rosensweig instability with Faraday instability is created, where the system is assumed to be stressed by a normal alternating magnetic field together with an external vertical oscillating force. First, it is assumed that the liquid is inviscid and thereby the motion is irrotational, a system of nonlinear coupled evolution equations governing the complex amplitudes of the different modes is derived. Second, a system of linear equations, derived via solving the linearized Navier–Stokes equations, is obtained. Consequently, the nonlinear equations of the complex amplitudes that correspond to the ideal fluid case are modified by adding the linear damping. This system is exploited to determine the steady-state solutions and then studying their stability both analytically and numerically. The results show that the liquid viscosity rather than the magnetic field affects the qualitative behavior of the wave motion and the system response alternates between the regular periodic and chaotic behavior depending on the specific values of some parameters.

1. Introduction

Nearly all of the liquid-filled basins with which we are familiar, ranging from the common cup of tea to the huge rocket propellant, the common feature is that this liquid has a free surface. The motion of liquid with a free surface is of great concern in many engineering disciplines such as fuel sloshing of the rocket propellant, oil oscillation in large storage tanks, water oscillation in a reservoir due to earthquake, sloshing of water in pressure-suppression pools of boiling water reactors and several others. This surface is subjected to the so-called parametric instability as the base flow of the liquid system is periodic in time. This kind of instability was first discovered by Faraday [1], when he immersed a vibrating plate in water. Specially, he noted that the subharmonic waves may be resulted on the free surface. However, the explanation of that phenomenon was provided in more detail by Benjamin and Ursell [2]. They observed that a vertically oscillating system is equivalent to a stationary system in the presence of a periodic gravitational field.

The nonlinearity in the equations of motion and boundary conditions governing any wave motion leads to wave interactions. However, by convection the linear terms may be put on the left-hand sides and the nonlinear terms on the right-hand sides of the equations and boundary conditions. Then the nonlinear terms may be regarded as small terms forcing the linear system. Phillips [3] was the first to consider weakly nonlinear interactions of waves, finding that interactions of surface gravity waves are enhanced when certain resonance conditions are satisfied. Since 1960, the theory has been developed and refined by Phillips and many other authors; also the theory has been applied to many kinds of waves in

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fluids. An example of a parametric instability driven by nonlinear interactions is the presence of edge waves on sloping boundaries. These trapped modes propagate in the longshore direction and were shown, by Guza and Davis [4], to be resonated by weakly nonlinear, normally incident surface waves. They concluded that edge waves play an important role in the development of regularly spaced beach cusps in many coastal areas. Feng and Sethna [5] have considered surface waves motions in a container with a nearly square base subjected to a vertical oscillation. Their treatment, based on perturbation expansions, emphasis on the symmetry-breaking aspect of the problem rather than classification of bifurcation sequences which occur when the symmetry is preserved. They have chosen just one example in a parameter range which covers the case of finite fluid depth [5]. Nagata [6] has investigated the stability properties of the surface wave motions at the interface of an inviscid fluid within a container with a square base which is subjected to a vertical oscillation when the amplitude of the oscillation is small and the frequency of the oscillation is close to twice the natural frequency of the system. The stability of the flat surface and the descriptions of the single solutions as well as the mixed modes are analyzed. Faraday resonance of a monochromatic interfacial waves in a two-layer, weakly-viscous system in a rectangular basin has been investigated by Hill [7]. Faraday waves occur on the surface of a liquid (with frequency ω) when the container undergoes small vertical sinusoidal oscillations with frequency 2ω (see [8–10] for an overview) where nonlinear amplitude equations are often used to model these waves. Some research that deals with rectangular containers was done by Huntley [11] and Mahony and Smith [12]. In [12], the deep water problem for a rectangular organ pipe is considered. The flow field was assumed to be two-dimensional. It was shown that surface water waves may be excited by high-frequency acoustic fields. More recently, Yoshimatsu and Funakoshi [13] investigated the resonance waves in square containers caused by horizontal oscillations. It was shown that the kind of waves observed depends on the angle of the direction of oscillation with a container wall. A weakly nonlinear theory [14] has been developed by an asymptotic approach for excited capillary-gravity waves in a water filled two-dimensional rectangular basin under some edge condition at the contact line. In that study, they assumed that the forcing frequency is near a resonance frequency or some combination of the resonance frequencies. By using a two time-scale asymptotic expansion of the solution and solvability conditions for the equations of the third-order approximations in the expansion, the amplitude equations of the excited surface waves at the resonance frequencies are derived.

The role of resonant triad interactions in the formation of Faraday wave patterns has been investigated extensively by Viñals and co-workers for both the case of single frequency forcing [15,16], and two-frequency forcing [8]. The most detailed two-frequency calculations focused on the situation where the frequency ratio was 1/2, and the onset surface wave response was subharmonic with the forcing. Zhang and Viñals [16] compared their theoretical results with experimental results of Müller [17], who observed subharmonic hexagons, triangles and squares near the bicritical point; which pattern was observed depended on a relative phase between the ω and 2ω sinusoidal waveforms in the forcing function. A key theoretical idea behind the work of Zhang and Viñals [16,18], and Chen and Viñals [15] is that the presence of certain resonant triads composed of Fourier mode wave vectors $\mathbf{k}_1$, $\mathbf{k}_2$ and $\mathbf{k}_1+\mathbf{k}_2$ can suppress the formation of regular wave patterns that involve the $\mathbf{k}_1$ and $\mathbf{k}_2$ modes. In particular, this is the case when the $\mathbf{k}_1$ and $\mathbf{k}_2$ modes are excited at the onset of the Faraday wave instability, while the mode with wave vector $\mathbf{k}_1 + \mathbf{k}_2$ is only weakly damped.

Magnetic fluids (or ferrofluids) are colloidal dispersions of single domain nanoparticles in carrier liquid. The attractiveness of ferrofluids stems from the combination of a normal liquid behavior with the sensitivity to magnetic fields. This enables the use of magnetic fields to control the flow of the fluid, giving rise to a great variety of new phenomena and to numerous technical applications [19]. One of the most interesting phenomena of pattern formation in ferrofluids is the Rosensweig instability [20]. At a certain intensity of the normal magnetic field the initially flat surface of a horizontal ferrofluid layer becomes unstable. Peaks appear at the fluid surface, which typically form a static hexagonal pattern at the final stage of the pattern-forming process [21]. By including vertical vibrations to that setup, Müller [22] analyzed the Faraday instability of viscous ferrofluids subjected to a vertical magnetic field in the frame of a linear stability analysis. Müller has concluded that the joint action of the two destabilizing factors leads to a delay of the Rosensweig instability. Moreover, it is found that by an appropriate choice of the system parameters, one can observe either a normal or anomalous dispersion. Parametric waves on the surface of a ferrofluid that excited by a vertical alternating magnetic field are investigated by Browaeys et al. [23]. This phenomenon is called the magnetic Faraday instability. In [23] the dispersion relation was investigated experimentally and it displays two significant features. The response of the surface waves is harmonic with respect to the frequency of the magnetic field and the wave vector of the resulting rolls is parallel to the field and consequently the crests and toughs of the rolls are perpendicular to the field. Mekhonoshin and Lang [24] have investigated the linear stability of the flat interface of an unbounded layer of ferrofluid of arbitrary depth in the presence of vertical vibrations and a horizontal magnetic field. A nonmonotonic dependence of the stability threshold on the magnetic field is found at high frequencies of the vibrations. They concluded that the horizontal magnetic field tends to decrease the curvature of the ferrofluid surface along the direction of the field. In other words, the horizontal field tends to stabilize the flat surface.

Elhefnawy et al. [25] studied the nonlinear stability of magnetized standing waves on the plane interface separating two immiscible inviscid magnetic fluids in a cylindrical container in the presence of both a uniform magnetic field and a periodic acceleration normal to the free surface. It found that the modulation of the amplitudes and phases of the two-modes resonant waves are governed by a system of nonlinear first order differential equations which in turn utilized to determine the steady state solutions and consequently investigating their stability. It is concluded that the frequency-response curves exhibit the transcritical and Hops bifurcations.

In this work, we have studied theoretically the nonlinear parametric instability of three-mode resonated standing waves raised at the interface of a bounded layer of a viscous magnetic fluid that excited by both alternating magnetic field and