RELATIVISTIC MODELLING OF STABLE ANISOTROPIC SUPER-DENSE STAR

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In the present article we have obtained new set of exact solutions of Einstein field equations for anisotropic fluid spheres by using the Herrera et al. [1] algorithm. The anisotropic fluid solutions so obtained join continuously to the Schwarzschild exterior solution across the pressure-free boundary. It is observed that most of the new anisotropic solutions are well-behaved and are used to construct the super-dense star models such as neutron stars and pulsars.

Keywords: anisotropic fluids, anisotropic factor, Einstein's equations, Schwarzschild solution, neutron stars, pulsars.

1. Introduction

The first ever exact solution of Einstein's field equations for a compact object in static equilibrium was obtained by Schwarzschild in 1916. The static isotropic and anisotropic exact solutions describing stellar-type configurations have continuously attracted the interest of physicists like Herrera et al. [1, 41]. Tolman [2] has proposed an easy way to solve Einstein's field equations by introducing an additional equation necessary to give a determinate problem in the form of some ad hoc relation between the components of metric tensor. According to this methodology, he obtained eight solutions of the field equations, and his important approach still is used in obtaining the exact interior solutions of the gravitational field equations for fluid spheres. Buchdahl [3] proposed a famous bound on the mass radius ratio of relativistic fluid spheres as $2GM/c^2r \leq 8/9$, which is an important contribution in order to study stability of the fluid spheres. Also Ivanov [4] has given the upper bound of the red shift for realistic anisotropic star models which cannot exceed the values of 3.842, provided the tangential pressure satisfies a strong energy condition $(\rho \geq p_r + 2p_t)$ and when the tangential pressure satisfies the dominated energy condition $(\rho \geq p_t)$.

Buchdahl [3] also obtained a nonsingular exact solution by choosing a particular choice of the mean density inside the star.

The theoretical investigations of realistic fluid models indicate that stellar matter may be anisotropic at least in certain density ranges ($\rho > 10^{15}$ gm/cm^3) (Ruderman [5] and Canuto [6]) and radial pressure may not be equal to the tangential pressure of stellar structure. The existence of a solid core due to the presence of anisotropy in the pressure was thought of for type-3A super-fluid (Kippenhahm and Weigert [7]), different form of phase transitions (Sokolov [8]) or for other physical phenomena. On the scale of galaxies, Binney and Tremaine [9] have considered anisotropies in spherical galaxies, from a purely Newtonian point of view. The mixture of two gases (e.g. ionized hydrogen and electrons or monatomic hydrogen) can be described formally as an anisotropic fluid (Letelier [10] and Bayin [11]). The importance of equations of state for relativistic anisotropic fluid spheres have been investigated by generalizing the equation of hydrostatic equilibrium to include the effects of anisotropy (Bowers and Liang [12]). Their study shows that anisotropy may have nonnegligible effects on parameters such as maximum equilibrium mass and surface red shift. The relativistic anisotropic neutron star models with high densities by means of several simple assumptions showed that there is no limiting mass of neutron stars for arbitrary large anisotropy which have been studied by Heintzmann and Hillebrandt [13]. However, maximum mass of a neutron star still lies beyond 3–4 $M_\odot$. Also, the solutions for an anisotropic fluid sphere with uniform density and variable density have been studied by Maharaj and Maartens [14] and Gokhroo and Mehra [15], respectively. Most of the astronomical objects have variable density. Therefore, interior solutions of anisotropic fluid spheres with variable density are more realistic physically.

Many workers have obtained different exact solutions for isotropic and anisotropic fluid spheres in different contexts (Delgaty and Lake [16], Dev and Gleiser [17], Komathiraj and Maharaj [18], Thirukkanesh and Ragel [19], Sunzu et al. [20], Harko and Mak [21], Mak and Harko [22], Chaisi and Maharaj [23], Maurya and Gupta [24–26], Feroze and Siddiqui [27], Pant et al. [28–30], Bhara et al. [31], Monowar et al. [32], Kalam et al. [33], Consenza et al. [34], Krori [35], Singh et al. [36], Patel and Mehta [37], Malaver [38, 39], Escupli et al. [40], Herrera and Santos [41, 42], Herrera et al. [43, 44]).

The present paper consists of nine sections, Section 1 contains introduction; Section 2 contains metric, its components and the field equations. Section 3 embodies the solutions of anisotropic fluid spheres in different contexts. Section 4 contains the expressions for density and pressure which are mentioned for each fluid sphere. Section 5 describes various physical conditions to be satisfied by the anisotropic fluid spheres. The analytical behaviours of solutions under physical conditions (mentioned in Section 5) are mentioned in Section 6. Section 7 describes the evaluation of arbitrary constants involved in the fluid solutions by means of the smooth joining of Schwarzschild metric at the pressure-free interface $r = a$. The stability of models is proposed in Section 8, and finally Section 9 includes the physical analysis of the solutions so obtained along with the concluding remarks.