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Conserved quantity of elastic waves in multi-layered media: 2D SH case — Normalized Energy Density

Hiroyuki Goto [a,*], Sumio Sawada [a], Toshiyuki Hirai [b]

[a] Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan
[b] NEWJEC Inc., 2-3-20 Honjo-Higashi, Kita-ku, Osaka 531-0074, Japan

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ABSTRACT

We introduce a new conserved quantity, Normalized Energy Density (NED), alternative to the conventional definition of energy for a layered structure in a 2D SH problem. NED is defined by the average of power of a half transfer function multiplied by the impedance, and the conservation across the material interface is analytically proved for a two-layered case. For three, four, and ten-layered cases, the conservation is examined by applying the Monte Carlo simulation method, and then NED is supposed to be conserved through the layers.

1. Introduction

Conserved quantities, such as mass, momentum and energy, in elasto-dynamic problems are the fundamental variables when analyzing wave propagation in a continuous medium. In addition, the balance principles associated with these quantities, e.g., the balance of mass and the balance of momentum, govern the deformation within the framework of Newtonian mechanics. The balance of energy is one of the principles used to quantify the seismic energy radiated from an earthquake source.

Radiation energy E is theoretically defined as the total energy transmitted through a certain surface, S, as follows:

$$E = -\int_0^\infty dt \int_S \left(\sigma_{ij} - \sigma_{ij}^0\right) \dot{u}_i n_j dS, \tag{1}$$

where σ_{ij} and σ_{ij}^0 are the tentative and the initial stress tensors, respectively. $\dot{u}_i$ is the particle velocity, and n_j is a normal vector of the surface S. When a particular region, e.g. a seismic fault, generates all of the energy, the integration on the arbitrary surface S surrounding the region is theoretically conserved even for a general heterogeneous medium. The above representation has already been introduced by Love [1]. The energy of seismic events was first applied by Richter [2] in order to measure the size of earthquakes by using the local magnitude scale (M_L), although it was not exactly equal to the definition of the energy. Afterward, [3] proposed the use of moment magnitude (M_W), defined from the seismic moment that is related to the energy release during the events, whose energy is different from the radiation energy (Eq. (1)). A detailed discussion on radiation energy is introduced by Kostrov and Das [4], Fukuyama [5], and Abercrombie et al. [6].

If a seismic wave through the surface S is approximated by a single plane wave, either a P- or an S-wave propagated in a uniform direction, the energy for the P-wave case, E_α, and that for the S-wave case, E_β, are represented as follows:

$$E_\alpha = \int_0^\infty dt \int_S \rho\alpha \dot{u}_\alpha^2 l_i n_i dS, \quad E_\beta = \int_0^\infty dt \int_S \rho\beta \dot{u}_\beta^2 l_i n_i dS, \tag{2}$$

* Corresponding author. Tel.:+81 774 38 4067; fax: +81 774 38 4070.
E-mail address: goto@catfish.dpri.kyoto-u.ac.jp (H. Goto).

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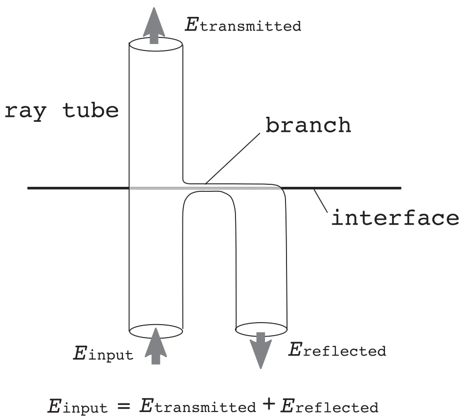

Fig. 1. Ray tube at the material interface and the energy conservation.

where ρ, α and β are the density, the P-wave velocity, and the S-wave velocity, respectively. $\dot{u}_\alpha$ and $\dot{u}_\beta$ are the amplitudes of particle velocity for the P-wave and the S-wave, respectively. l_i is a vector representing the direction of the wave propagation. The energy density, defined by the integrand, is a product of the square of the particle velocity and the impedance. Note that the total energy for a general wave field, represented by the superposition of the P- and the S-waves, is not equal to $E_\alpha + E_\beta$ (see Appendix A).

A part of the energy integrated on the shrunken area of S is utilized as a principle of energy conservation when all of the input energy is confined in a certain region, so-called "ray tube" [7]. The energy on the cross-sectional area of the tube is theoretically conserved. Here, we focus on the layered structure. At the interface, part of the energy for the input wave is transmitted, and the rest is reflected. Then, both the transmitted and the reflected waves should be considered in order to apply the energy conservation in the ray tube. As shown in Fig. 1, the sum of the transmitted energy and the reflected energy is equal to the input energy. However, the total input energy cannot be observed in only the opposite layer because the transmitted energy is part of the input energy. Therefore, the energy is not conserved across the interfaces. Note that some researchers apply the energy, directly defined by $\int_0^\infty \rho c \dot{u}^2 dt$, to the layered structure (e.g., Kokusho and Motoyama [8]), however, they do not pay attention to the fact that the quantity is not conserved. If a quantity conserved over the layer structure exists, absorbed energy in propagating in the layer might be estimated, directly. The quantification of the absorbed energy helps to understand the hysteretic damping due to anelasticity, e.g. Q-factor, and the soil nonlinearity, as discussed by Kokusho and Motoyama [8].

In this article, we introduce a quantity, Normalized Energy Density, which is an alternative to the conventional definition of energy, and discuss the features of the 2D SH problem. The quantity is analytically discussed for the two-layered case, and numerically examined for multi-layered cases.

2. Two-layered case

The theoretical implementation starts from the waves, vertically propagated into a simple two-layered structure. Only 2D SH waves, which have an antiplane amplitude with respect to the plane, are considered here. The structure consists of a horizontal layer, Layer #1, with a thickness of h and a half space basement, Basement #0. The S-wave velocity and the density are β_1 and ρ_1 for Layer #1 and β_0 and ρ_0 for Basement #0, as shown in Fig. 2. An incident plane wave propagates vertically into Layer #1 through the interface between Layer #1 and Basement #0. Each layer keeps elasticity independent of the wave amplitude.

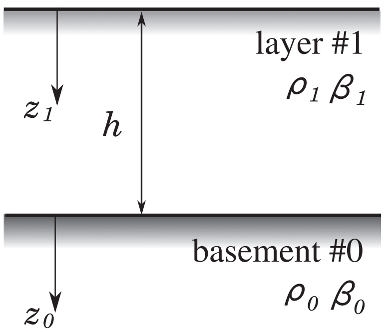

Fig. 2. Two-layered model.