Wave Motion 48 (2011) 681–700

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Generalized impedance boundary conditions for thin dielectric coatings with variable thickness

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ARTICLE INFO

Article history:
Received 15 October 2010
Received in revised form 5 March 2011
Accepted 10 June 2011
Available online 21 June 2011

Keywords:
Scattering problems
Electromagnetism
Asymptotic models
Generalized impedance boundary conditions
GIBC Thin coatings

ABSTRACT

We derive the so-called Generalized Impedance Boundary Conditions (GIBC) that model thin dielectric coatings with variable width located on a perfect electric or magnetic conducting surface. We treat the 2-D electromagnetic problem for both TM and TE polarizations. The expressions of the GIBCs are obtained up to the third order (with respect to the coating width). The order of convergence is numerically validated through various numerical examples using a finite element type method. Particular attention is given to the cases where the inner boundary has corner singularities.

1. Introduction

Effective Impedance boundary conditions (EIBC) are widely used to simplify the mathematical and numerical complexities in the solution of scattering problems in electromagnetic theory and have been the subject of several research activities [1–6]. EIBC have been developed for such objects as the earth surface, thin layers of dielectrics, multilayered dielectric structures [7–9]. The surface impedance is, in general, a tensor, and it may be an inhomogeneous scalar under an isotropic assumption. The determination of EIBC for a given scatterer constitutes an important class of problems in the electromagnetic theory, and various approximate methods have been established in the literature for special kinds of geometries and surfaces [10–15]. In all these methods, one first tries to solve the direct scattering problem for a given scattering structure and, then, express the EIBC in terms of the electric and magnetic field on the boundary.

A class of EIBC is known as Generalized Impedance Boundary Conditions (GIBC) [16–24] associated with scattering problems from coatings of small thickness or conductive thin sheets. The expression of these conditions are independent of the incident wave and apply to general geometries of the coated obstacles. They lead to approximate models that are close to the original one up to an $O(\delta^{k+1})$ error, where k denotes the order of the GIBC and δ is the thickness of the coating. They are numerically attractive since associated scattering problems are formulated only on the domain exterior to the coating, and therefore do not require a meshing of the thin layer.

The aim of this paper is to derive GIBC for the dielectric coatings with variable width. Along this line we consider time harmonic electromagnetic scattering problems related to coatings located on perfect electric or magnetic conducting bodies whose width δ is small compared to the wavelength of the incident field. Considering configurations where the coating can have a variable widths intends to widen applicative perspectives of GIBC (localized coatings, corrugated surfaces,…). These cases are more technical than the case of coatings with constant width (that can be found in the classical literature on the subject) and lead to non intuitive

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doi:10.1016/j.wavemoti.2011.06.002

expressions of the GIBC. We shall restrict ourselves in these first investigations to the 2-D problem but consider both possible polarizations of the incident electromagnetic wave: TM corresponding to a Dirichlet boundary condition on the perfect scatterer and TE corresponding to a Neumann boundary condition. In order to derive these expressions we adopted a formalism similar to [25,26] based on the so-called scaled asymptotic expansions. Semi-analytical expressions of the expansion are obtained for each polarization till the third order. Let us however emphasizes that while a notable difference can be observed in the derivation of the GIBC expressions, the theoretical justification of the obtained models (i.e. derivation of error estimates) would follow the same lines as in the constant case and for sufficiently regular geometries (see for instance [18]). We therefore shall only concentrate on the numerical validation of obtained models. With that perspective, a number of experiments are conducted to check the formally predicted convergence rate for the derived models. Note that the scattering problem related to GIBC model can be solved by one of the well-known numerical techniques. Here we present a finite element type solution and the details are also given. Particular attention is given to the case where the interior boundary of the coating is not regular (the outer boundary is however assumed to be regular). We explain how the expressions of the GIBC can be adapted to these cases and numerically test that the adaptation preserves the formally predicted rate of convergence for this case.

The document is organized as follows. The next section is devoted to a presentation of the mathematical model associated with the "exact" scattering problem and introduces the concept of GIBC together with some useful tools of differential geometry. The third section is dedicated to the derivation of GIBC expressions using the method of scaled asymptotic expansions. A finite element method is given for the solution of scattering problems related to objects having GIBC in the fourth section. Finally, the validation of the GIBC expressions is illustrated through several numerical examples in the last section.

2. Formulation of the problem

Consider the scatterer illustrated in Fig. 1. In this configuration a homogeneous, lossy, non-magnetic dielectric material denoted by Ω_+^δ is coated on a perfect electric conductor (PEC) or perfect magnetic conductor (PMC) denoted by Ω^δ, where δ represents the (variable) width of the coating. The exterior medium is the vacuum and is symbolized by Ω_-. In the mathematical analysis below we assume that the exterior medium is independent of δ and shall denote by Γ its boundary and by Γ^δ the boundary of Ω^δ. We denote by $\vec{n}$ (resp. $\vec{n}^\delta$) the unitary normal vector field defined on Γ (resp. Γ^δ) and directed to the exterior of Ω_- (resp. Ω_+^δ). Now we consider the time harmonic scattering problem related to this coated object: the total field u^δ is decomposed into u_-^δ and u_+^δ that respectively denote the restriction of u^δ into Ω_- and Ω_+^δ. These fields satisfy

$$\Delta u_-^\delta + k_0^2 u_-^\delta = 0 \quad \text{in } \Omega_- \tag{1}$$

$$\Delta u_+^\delta + k_1^2 u_+^\delta = 0 \quad \text{in } \Omega_+^\delta, \tag{2}$$

with the continuity conditions

$$u_-^\delta = u_+^\delta \quad \text{on } \Gamma, \tag{3}$$

$$\frac{\partial u_-^\delta}{\partial n} = \frac{\partial u_+^\delta}{\partial n} \quad \text{on } \Gamma, \tag{4}$$

where $k_0 : = \omega\sqrt{\varepsilon_0\mu_0}$ and $k_1 : = \sqrt{\omega^2\varepsilon_1\mu_0 + i\omega\sigma_1\mu_0}$ respectively denote the wave numbers inside and outside the coating, with ω being the frequency, ε_0 and ε_1 respectively denoting the dielectric permittivity of the background medium and the coating,

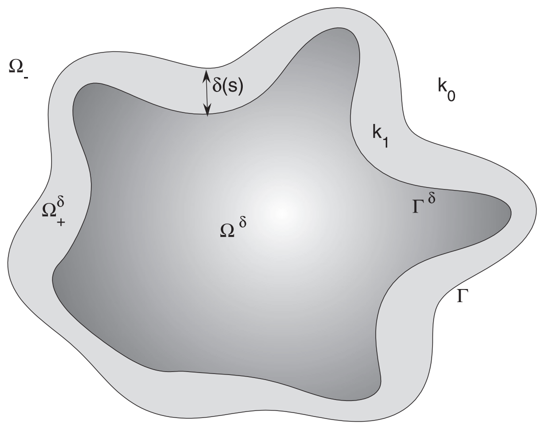

Fig. 1. Geometry of thin coating with variable width.