Wave Motion 48 (2011) 259–267

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Analytical study on long wave refraction over a dredge excavation pit

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ARTICLE INFO

Article history:
Received 18 March 2010
Received in revised form 8 September 2010
Accepted 2 November 2010
Available online 11 November 2010

Keywords:
Long wave
Dredge excavation pit
The mild-slope equation
Analytical solution

ABSTRACT

An analytical solution of the mild-slope wave equation is derived to describe long wave propagating over the idealized dredge excavation pit. The pit is assumed to be axisymmetrical and composed of a flat bottom and a convex slope. The convex slope is expressed by a simple power function. The problem is solved in the polar coordination system by the separation of variables. By the obtained solution, the characteristics of the wave refraction and reflection over the dredge excavation pit are discussed. The results show that wave amplitude is attenuated within and in the lee side of the pit and amplified at the rear flank of the pit due to wave refraction. The effects of the incident wave length and the shape of the pit on wave refraction are also discussed.

1. Introduction

The large volume of sand for land reclamation or for beach nourishment is, in many occasions, obtained by offshore dredging. Offshore dredging has long been considered to have minimum impacts on the environment, but it always leaves excavation pits, and recent studies tend to support that the effects of such a pit on the ecosystem are actually not negligible [6].

Wave is usually an important factor in coastal environmental assessment. A comprehensive understanding on wave refraction due to a dredge excavation pit may then be necessary. However, the relevant research efforts do not seem to have generated many straightforward references. Suh et al. [7] derived a solution for long waves propagating over a circular bowl pit. Recently, Jung and Suh [3] derived a new solution of the extended mild-slope equation for an axisymmetric pit, considering the effect of rapidly varying bathymetry.

The behavior of water waves over a varying bathymetry is an important topic in coastal engineering research. A great number of researches based on numerical modeling have been carried out, but analytical studies are rare. Among the limited number of valuable analytic solutions, a few are based on the mild-slope equation or its reduced form.

The mild-slope equation is well known to be a practically useful wave model, which can describe the behavior of water waves under combined diffraction and refraction [2,5]. Zhang and Zhu [9] presented two solutions of the equation on long waves around a conical island and over a paraboloidal shoal. Zhu and Zhang [10] also presented a solution for a cylinder mounted on a conical shoal. Yu and Zhang [8] derived a uniform solution for axisymmetrical bottom geometries, which is described by a power function in the radial direction, and applied to study wave scattering by a cylinder mounted on a circular shoal. With the long wave assumption eliminated, Lin and Liu [4] derived a solution of the mild-slope equation by an explicit approximation of the dispersion equation, and studied the case of the submerged truncated paraboloidal shoal. Zhu and Harun [11] obtained a solution for long wave over a circular hump.

In this study, we discuss the long wave refraction over a dredge excavation pit. The geometry of the pit is idealized to be axisymmetrical with a flat bottom and a convex slope. A matched general solution of the problem is sought. By this solution, the

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0165-2125/$ – see front matter
doi:10.1016/j.wavemoti.2010.11.001

characteristics of the long waves over the underwater pit are discussed in detail. The analytical solution is obtained under a simplified condition, but it is expected to be useful in understanding the physics and in testing numerical models.

2. Analytical solution

2.1. Theoretical background

The physical problem of our primary interest is shown in Fig. 1 with a cross-sectional view and a plane view. The dredge excavation pit is assumed to be axisymmetrical in the horizontal plane. The uniform water depth in the open sea is denoted by h_0, and the uniform water depth within the pit is denoted by h_1. The variable water depth over the slope is assumed to be expressed by $h(r)=\lambda r^{-\alpha}$, where r is the horizontal distance from the center of the pit and, α and λ are positive constants. r_1 is the radius of the flat bottom and r_0 is the maximal radius of the pit.

By assuming that the wave is of small amplitude, the slope of the pit is gentle, and the dissipation of wave energy can be neglected, we employ the mild-slope wave equation [2] to describe the wave motion. When the water surface elevation is expressed in the form of $\eta(x,y)e^{-i\sigma t}$, in which σ is the wave angular frequency and the complex-valued η with its modulus representing the wave amplitude and its argument representing the relative phase is also called the water surface elevation, the mild-slope wave equation can be written as

$$\nabla\cdot\left(CC_g\nabla\eta\right)+k^2CC_g\eta=0 \tag{1}$$

where ∇ represents del operator in the horizontal plane; C is the wave celerity, C_g is the wave group velocity, and k is the wavenumber. As we additionally adopt the long wave assumption, i.e., we assume that $C\approx C_g\approx\sqrt{gh}$, where g is the gravity acceleration, Eq. (1) can be further simplified as

$$\nabla\cdot(h\nabla\eta)+\left(\frac{\sigma^2}{g}\right)\eta=0 \tag{2}$$

where the water depth h depends on the horizontal position. With the polar coordinates r and θ, Eq. (2) can be rewritten as

$$r^2\frac{\partial^2\eta}{\partial r^2}+r\left(1+\frac{r}{h}\frac{\partial h}{\partial r}\right)\frac{\partial\eta}{\partial r}+\frac{1}{h}\frac{\partial h}{\partial\theta}\frac{\partial\eta}{\partial\theta}+\frac{\partial^2\eta}{\partial\theta^2}+\frac{\sigma^2r^2}{gh}\eta=0 \tag{3}$$

The third term in Eq. (3) can be omitted by considering that the bathymetry is axisymmetrical. By the separation of variables, we note

$$\eta(r,\theta)=\sum_{n=0}^{\infty}R_n(r)(C_{3n}\cos n\theta+C_{4n}\sin n\theta) \tag{4}$$

where C_{3n} and C_{4n} are constants, and $R_n(r)$ satisfies

$$r^2\frac{d^2R_n}{dr^2}+r\left(1+\frac{r}{h}\frac{\partial h}{\partial r}\right)\frac{dR_n}{dr}+\left(\frac{\sigma^2r^2}{gh}-n^2\right)R_n=0 \tag{5}$$

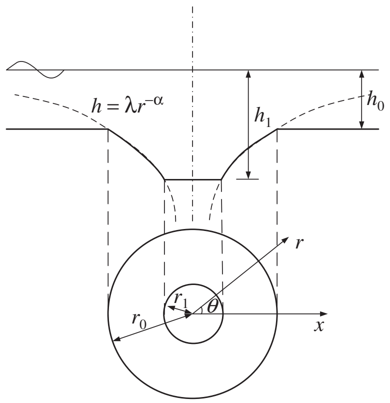

Fig. 1. Definition sketch of the physical problem.