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Acoustic scattering of a plane wave by a circular penetrable cone

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ABSTRACT

The paper is devoted to the diffraction of a plane wave by an acoustically transparent semi-infinite cone. The problem of diffraction is reduced to a singular integral equation in the framework of the incomplete separation of variables. The Fredholm property and regularization of the integral equation are discussed. Some important integral representations of the wave field are considered. The detailed study of the far-field asymptotics is given. Expressions for the diffraction coefficient of the spherical wave scattered from the vertex of the cone are considered. The reflected from the conical surface waves and those transmitted across the surface are also discussed.

1. Introduction

The problem of diffraction by a conical singularity is studied in the numerous papers (see the References). The different types of the boundary conditions on a conical surface were considered, which have required elaboration of the different approaches for the ideal boundary conditions (Dirichlet, Neumann or their electromagnetic analogues), e.g. [3–5,11–13,16,17,21,25,33,34] or for the impedance type conditions, e.g. [7–10,28–31]. However, the study of the scattering of waves by a conical surface separating two media with the different wave velocities c_1 (in the exterior) and c_2, (in the interior, $c_1 > c_2$) is, in some aspects, more complicated due to the variety of the studied wave processes. On the other hand, the methods and experience gained from the above-mentioned works play an important role for the problem at hand. To our knowledge, there are, at least, two papers [22,27][1] devoted to the diffraction by a penetrable cone.

The harmonic wave described by U^{inc} (acoustic pressure) is incident from the exterior Ω_1 of the semi-infinite conical surface C and, thanks to the refraction, penetrates into the interior Ω_2. In view of the diffraction phenomena the total field $U^{inc} + U$ is the sum of the incident and scattered waves ($e^{-i\omega t}$ time dependence is suppressed throughout the paper). The principal aim of our study is to describe the wave components in the far-field asymptotics of the scattered field for the diffraction problem at hand.

In the next section we formulate the problem. By means of the Kontorovich–Lebedev transformation it is reduced to a problem on a unit sphere after the separation of the radial variable. Further separation of the angular variables leads to a singular integral equation for the corresponding Fourier coefficients. We study the equation by considering its Fredholm property. Some useful integral representations for the wave field are then discussed and a formula for the diffraction coefficient of the spherical wave from the vertex of the cone is given for the part of the domain Ω_1 not illuminated by the reflected waves.

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[1] Unfortunately the important paper [22] was not known to the author of work [27].

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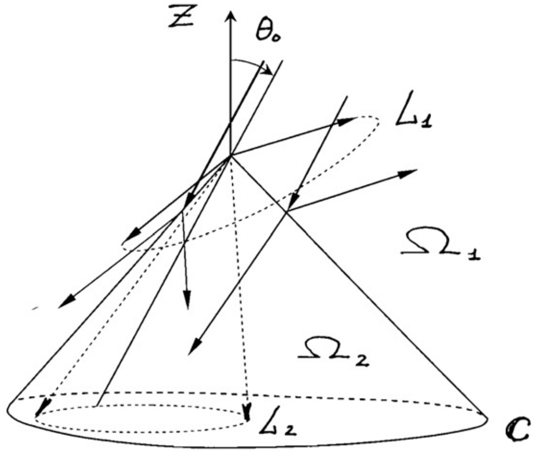

Fig. 1. Diffraction by a transparent cone.

Basing on the results of the first part of the present work, in the second part we carefully investigate different components of the acoustic wave field in the far-field asymptotics. We describe the waves reflected and transmitted (refracted) by the conical surface. The diffraction coefficient (scattering amplitude) of the spherical wave from the vertex of the cone is then additionally studied for the domains illuminated by the reflected or transmitted rays. Some comment are also given about the head wave which propagates in the medium with the lower wave velocity, i.e., in Ω_2.

In comparison with the results of the work [27], in the present paper we develop a modified procedure to derive the singular integral equation and study its Fredholm property. Moreover, we give an approach to investigate components of the far-field asymptotics, in particular, a new analytic expression for the diffraction coefficient of the spherical wave from the vertex of the cone is studied. The new developments require a flexible use of the Sommerfeld integral and careful studies of the analytic properties of the Sommerfeld transformant.

2. Formulation

The spherical coordinate system $r, \omega=(\theta,\varphi)$ is attached to the vertex of the conical surface C with the axis OZ coinciding with the axis of symmetry of the right-circular cone (Fig. 1). The equation of the conical surface C is given by $\theta=\theta_1$. For some technical simplifications we assume that $\pi/2<\theta_1\leq 3\pi/4$. The incident plane wave is given by the expression

$$U^{inc}(k_1 r, \omega, \omega_0) = e^{-ik_1 r\, cos\theta_i(\omega,\omega_0)}, \tag{1}$$

where $k_1=\omega/c_1$ is the wave number in Ω_1, ($k_2=\omega/c_2$ is the wave number in Ω_2), $\cos\theta_i(\omega,\omega_0)=\cos\theta\cos\theta_0+\sin\theta\sin\theta_0\cos[\varphi-\varphi_0]$ and $\omega_0=(\theta_0,\varphi_0)$ specifies the direction from which the incident wave comes from infinity.

The scattered fields $U(k_1r,\omega,\omega_0)$ in Ω_1 and $V(k_2r,\omega,\omega_0)$ in Ω_2 fulfill the Helmholtz equations

$$\left(\triangle + k_1^2\right)U(k_1 r, \omega, \omega_0) = 0, \qquad \left(\triangle + k_2^2\right)V(k_2 r, \omega, \omega_0) = 0, \tag{2}$$

where

$$\triangle = \frac{1}{r^2}\left(\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\triangle_\omega, \qquad \triangle_\omega = \frac{1}{sin\theta}\left(\frac{\partial}{\partial\theta}sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{sin^2\theta}\frac{\partial^2}{\partial\varphi^2},$$

$\triangle_\omega$ is the Laplace–Beltrami operator. It is useful to introduce the refraction index

$$N = \frac{k_2}{k_1} > 1.$$

Sometimes it is convenient to assume that $N=N'+iN''$ is complex with Im $N''>0$, which implies absorption of energy in Ω_2.[2]

[2] In the framework of this assumption uniqueness of the solution has been studied [27].