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Effect of axial load on the propagation of elastic waves in helical beams

Ahmed Frikha [a], Fabien Treyssède [a,*], Patrice Cartraud [b]

[a] Laboratoire Central des Ponts et Chaussées, 44341 Bouguenais Cedex, BP 4129, France
[b] GeM, UMR CNRS 6183, École Centrale de Nantes, 1 rue de la Noë, 44 321 Nantes Cédex 3, France

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ABSTRACT

Helical structures are designed to support heavy loads, which can significantly affect the dynamic behaviour. This paper proposes a physical analysis of the effect of axial load on the propagation of elastic waves in helical beams. The model is based on the equations of motion of loaded helical Timoshenko beams. An eigensystem is obtained through a Fourier transform along the axis. The equations are made dimensionless for beams of circular cross-section and the number of parameters governing the problem is reduced to four (helix angle, helix index, Poisson coefficient, and axial strain). A parametric study is conducted. The effect of loading is quantified in high, medium and low-frequency ranges. Noting that the effect is significant in low frequencies, dispersion curves of stretched and compressed helical beams are presented for different helix angles and radii. This effect is greater as the helix angle increases. Both the effects of stress and geometry deformation are shown to be non-negligible on elastic wave propagation.

1. Introduction

Helical structures are used in many engineering applications. Typical examples are helical springs, widely used in automotive and aeronautic industry, and steel multi-wire cables, largely encountered in civil engineering. These structures are usually subjected to large loads.

For the design of helical springs, several studies have been conducted to understand the dynamic behaviour and calculate the first vibration modes. First without considering the effect of applied loads, the computation of vibration modes of helical beams with circular cross-section has been performed based on analytical but approximate solutions [1], the finite element methods [2] or the assumed mode method [3] for instance. Another approach is the transfer matrix method, employed in [4,5]. An efficient numerical method for predicting the natural frequencies of helical springs has been developed in [6]. The dynamic stiffness method has been used by Pearson and Wittrick [7] to find an exact solution for vibration of helical springs with the Euler–Bernoulli model. Lee and Thompson [8] used the same method, but with the Timoshenko beam model.

However, the first vibration modes of helical springs correspond to low-frequency motions, which are strongly affected by the presence of applied axial loads. The vibration analyses of springs have hence been extended to account for load effects on the natural frequencies thanks to the finite element method [9], the dynamic stiffness matrix [8] or the transfer matrix method, used in [10,11]. However, as noticed in [12,17], Pearson's equations [10] do not reduce to equations for simpler rods when load terms are included. All these studies show the importance of considering axial loads, compressive in the analyses, for the computation of natural frequencies.

As far as elastic wave propagation is concerned, the literature on helical waveguides is rather scarce. An analytical beam model [13] as well as more general numerical approaches [14,15] has been recently proposed. In [16], a semi-analytical finite element method has also been proposed for the analysis of guided wave propagation inside multi-wire helical waveguides,

* Corresponding author. Tel.: +33 240845932; fax: +33 240845997.
E-mail address: fabien.treyssede@lcpc.fr (F. Treyssède).

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typically encountered in civil engineering. However, these studies neglect the presence of applied loads, whose effect remains unexplored on guided waves.

The aim of this paper is to investigate the effect of axial loads on the propagation of guided modes in helical waveguides. For simplicity, multi-wire waveguides are not considered and the model is based on the equations of motion of Timoshenko loaded helical beams. Such a model is not valid at high frequencies, when high order modes become propagating, but constitutes a first step and can serve as a reference solution before the development of fully three-dimensional models, as done in [14–16] without loads. A space Fourier transform along the helical axis is performed, yielding a wave propagation eigensystem whose zero determinant corresponds to the dispersion relationship. The equations are made dimensionless for beams of circular cross-sections. The problem is then governed by four parameters, which are the helix angle, the dimensionless radius (helix index), the dimensionless axial load (axial strain) and the Poisson coefficient.

The applied loads act on the dynamics through two effects: the deformation of the geometry and the stress generated inside the structure. Both effects are included in the present analysis. The deformed helix parameters are calculated using a non-linear model.

A parametric study is conducted in order to highlight the effect of axial loads, compressive or tensile, on waves for various helix angles and radii. Three frequency ranges are distinguished. A branch identification of dispersion curves is given for a better physical understanding of the different modes existing in helical waveguides. The effects of stress and deformation are also compared.

2. Model

2.1. Equations of motion for dynamics

One considers a helical beam with a circular cross-section of radius r. The helix centreline is defined by its pitch angle α_0 and radius R_0 in the unloaded state. In the loaded state, the spring is subjected to a static axial force P and the pitch angle and radius become α and R respectively (see Fig. 1). The curvature κ and torsion τ are given by $\kappa = \cos^2\alpha/R$ and $\tau = \sin\alpha\cos\alpha/R$. The Serret–Frenet basis $(\mathbf{e}_n, \mathbf{e}_b, \mathbf{e}_t)$ associated with the helix is shown in Fig. 1, where $\mathbf{e}_n$, $\mathbf{e}_b$ and $\mathbf{e}_t$ respectively denote the normal, binormal and tangent unit vectors. In this coordinate system, the static force is written as $[0,\ P\cos\alpha,\ P\sin\alpha]$ and the static moment as $[0, -PR\sin\alpha,\ PR\cos\alpha]$. P is taken positive when tensile.

In the framework of Timoshenko beam theory, the general equations governing the small perturbations of a helical beam subjected to a static axial load P are given by the following set of 12 equations which relate the forces and moments to the displacements and rotations [11,12,17]:

$$\frac{du_n}{ds} = \tau u_b - \kappa u_t + \phi_b + \frac{Q_n}{GA_n}, \tag{1}$$

$$\frac{du_b}{ds} = -\tau u_n - \phi_n + \frac{Q_b}{GA_b}, \tag{2}$$

$$\frac{du_t}{ds} = \kappa u_n + \frac{Q_t}{EA_t}, \tag{3}$$

$$\frac{d\phi_n}{ds} = \tau\phi_b - \kappa\phi_t + \frac{M_n}{EI_n}, \tag{4}$$

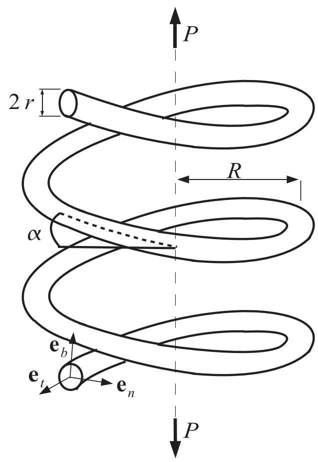

Fig. 1. Helical spring under axial load.