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# Surface waves at the interface between an inviscid fluid and a dipolar gradient solid

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## HIGHLIGHTS

- Dispersion of surface waves propagating at a fluid–solid interface is investigated.
- The solid is modelled as dipolar and second gradient continuum.
- Leaky Rayleigh and Scholte–Stoneley type solutions are investigated.
- A subsonic Leaky wave solution is observed and discussed.

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ABSTRACT

This paper is about the dispersion analysis of surface waves propagating at the interface between an inviscid fluid and a higher gradient homogeneous elastic solid modelled as a dipolar gradient continuum. In order to compare the results, a second gradient model is also evaluated. The analysis is carried out by finding the roots of the secular equation, and by carefully studying their physical meaning. As it is well known, higher gradient continua are dispersive, *i.e.* phase and group velocities are frequency dependent. As a consequence, the existence of surface waves will indeed depend on frequency. In order to investigate the behaviour of surface waves in this specific fluid–solid configuration, a complete dispersion analysis is performed, with a particular focus on the frequency range in which the phase velocity of shear waves is lower than the speed of waves of the fluid. Surface waves of the type Leaky Rayleigh and Scholte–Stoneley are observed in this frequency range. This work extends the knowledge on surface waves in the case of higher gradient solids and applications of these results can be found in the field of non-destructive damage evaluation in micro structured materials, composites, metamaterials and biological tissues.


## 1. Introduction

The study of surface waves propagating at the interface between a solid and a fluid is a topic of major interest in mechanics, (see Carcione [1] and Parker and Maugin [2]). Indeed, surface waves are frequently used in non destructive characterization of material properties, and their application spans from man made materials to biological tissues.

The aim of this work is then to investigate the behaviour of surface waves in the case of *solid–liquid interface*, when the solid is modelled as a higher gradient (or strain gradient) continuum. Two models are evaluated: the dipolar gradient and the second gradient. Higher gradient continua are a particular class of generalized continua, in which the energy of the

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system is supposed to depend on gradients of the displacement higher than the first (see Germain [3]). These models are mostly suitable to describe the static and dynamic behaviour of micro-structured materials at the edges of validity of the continuum model, *i.e.* when the wavelength of the perturbation, or the size of the specimen in case of static tests, approaches the characteristic length of the micro-structure. For a complete review of these theories, we suggest the recent work by Maugin and Metrikine [4].

While these theories have been available for decades, wave propagation in dipolar and second gradient continua has recently received a revival of attention. Several works on the subject can be found in the recent literature, see *e.g.* dell'Isola et al. [5], Gourgiotis et al. [6], Madeo et al. [7] and Rosi et al. [8]. This revival can be partially explained by the fact that the recent advances in meta-material synthesis and conception allow to produce artefacts that have the actual behaviour of a generalized continuum. Moreover, some experimental studies show that generalized models are also suitable to describe biological tissues as the cortical bone (see *e.g.* Ben-Amoz [9], Fotiadis et al. [10] and Protopappas et al. [11]).

Concerning the analysis of surface waves in dipolar gradient materials, only few studies are available in the literature, showing the dispersive behaviour of these waves (see *e.g.* Georgiadis et al. [12], Gourgiotis et al. [6]). However, all these works studied surface waves propagating at free interfaces. To the authors' knowledge, this is the first paper that is devoted to the case of the interface between a fluid and a higher gradient elastic solid.

One of the most interesting features of higher gradient models is that they are able to account for dispersive phenomena, *i.e.* phase and group velocities are frequency dependent. This is of great interest since a dispersive behaviour can be experimentally observed in a variety of actual microstructured materials, *e.g.* in bones (see Haïat et al. [13,14]). The fact that the phase velocities are frequency dependent entails important consequences on the existence conditions and propagation properties of surface waves.

Given that a common set-up in non destructive health monitoring tests involves a *solid–liquid interface*, this will be the configuration of our choice. It is well known that in the case of a *solid–liquid interface*, the existence of specific types of surface waves depends on the phase velocities of waves propagating in the considered media. In particular, since in the solid the velocity of bulk compressional (P-) waves is always higher than that of shear (S-) waves, two main configurations are usually studied, depending on the ratio $c_f/v_S$ between the speed $c_f$ of bulk waves in the fluid and the phase velocity $v_S$ of S-waves in the solid. The more conventional configuration, so called *hard solid–liquid interface*, is characterized by a ratio $c_f/v_S < 1$. In this case, the identification of surface waves is unique and well presented in the literature, in particular, a complete study can be found in Viktorov [15] and the references therein. Indeed, in that case the study of the secular equation permits the identification of two kind of surface waves: Scholte–Stoneley waves, which are vanishing away from the interface both in the fluid and the solid and whose velocity is real, and Leaky Rayleigh waves, which are vanishing only in the solid and whose velocity is complex, due to the energy leaking into the fluid.

The other configuration, the so called *soft solid–liquid interface*, characterized by a ratio $c_f/v_S > 1$. While no existence condition is posed for Scholte–Stoneley waves, it is a widely diffuse belief that this configuration does not allow for the existence of Leaky Rayleigh wave. This statement has been disproved in several papers. In Mozhaev and Weihnacht [16], it was showed that for a silver/gold alloy the threshold for the existence of Leaky Rayleigh waves is $c_f/v_S > 1.0145$. Moreover, the identification of the type of waves that exist above this threshold is difficult, as the roots of the secular equation, using a mathematical formalism, pass from one Riemann sheet to another and the corresponding solutions are not always physical. A complete study of all the roots of the problem is presented in Ansell [17], and gives a good picture of the complexity of the problem. Some studies are devoted to the study of existence of Leaky Rayleigh waves in this particular configuration, and the results are not always in agreement. In their work, Padilla et al. [18] demonstrate the existence of a Leaky Rayleigh wave whose phase velocity is close to $v_S$, while the same solution is found to be unphysical by Glorieux et al. [19].

The paper is organized as follows: after this introduction (Section 1), in Section 2 the governing equations for a fluid and both dipolar gradient and second gradient continua are recalled; in Section 3 the wave solutions for these equations are presented; Section 4 is devoted to the dispersion analysis and it is in this section that the main results will be presented. Finally, in Section 5 some conclusions are drawn.

## 2. Governing equations and boundary conditions

The objective of this section is to present the governing equations for the proposed system. Here, and throughout the paper, the presentation will be mainly focused on the dipolar gradient model. The second gradient model, as it will be considered as a limit case of the dipolar gradient one, will be presented at the end of each subsection.

### *2.1. Geometry and kinematics*

The geometry under consideration is composed by an upper half-space filled with an idealized acoustic fluid and a lower half-space filled with an homogeneous dipolar gradient or second gradient elastic solid. The two half-spaces are separated by a plane interface characterized by a normal vector $\mathbf{n} = (0, 1, 0)$ in a Cartesian coordinate system $(x_1, x_2, x_3)$ associated with the reference Cartesian frame $\mathbf{R}(O; \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ for which $O$ is the space origin and $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is an orthogonal basis, as depicted in Fig. 1. The state of the continua is described, in each part, by the three components of the displacement $u_i^{s,f}$ (for $i = 1, 2, 3$). We will use the superscript $s$ for the quantities defined in the solid and $f$ for the quantities defined in the fluid, where an ambiguity can occur.