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Feasible fundamental solution of the multiphysics wave equation in inhomogeneous domains of complex shape

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HIGHLIGHTS

- We consider the multiphysics wave equation in arbitrary inhomogeneous domains.
- We introduce new integral absorption condition at the boundary of a homogeneous domain.
- We derive feasible fundamental solution in the homogeneous domain using new condition.
- Feasible fundamental solution in the inhomogeneous domain accounts for the above solution.
- We determine surface and volume propagation integrals with the feasible kernels.

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ABSTRACT

Fundamental solutions of the linear equations governing mechanical and electromagnetic oscillations are kinematically represented by delay time along ray trajectories. The fundamental solutions can contain components which are not physically justified, if their ray trajectories are partly located outside the actual medium in accordance with Fermat's principle. To exclude all non-physical components and consider only the physically feasible fundamental solution, ray trajectories and delay time must satisfy the generalized Fermat's principle, as introduced by Hadamard in 1910. We introduce a rigorous dynamic description of this feasible fundamental solution satisfying the generalized Fermat's principle and being physically justifiable. The description is based on an integral condition of absolute absorption at the boundary of an effective medium. This condition selects a subset of the physically feasible fundamental solutions. We prove that, in homogeneous domains, the feasible fundamental solution is the sum of Green's function for unbounded medium and an operator Neumann series describing cascade diffraction at the boundary. In inhomogeneous domains we represent the feasible fundamental solution by an equation with a volume integral operator. The integral kernel contains the feasible fundamental solution for a homogeneous domain. We introduce feasible surface and volume integral operators that eliminate the unfeasible wavefields in the geometrical shadow zones.

1. Introduction

Fundamental solutions of the linear equations governing mechanical and electromagnetic oscillations are key elements of the mathematical theory of wave propagation. It is theoretically known that fundamental solutions are defined ambiguously and contain an arbitrary term which cannot be justified by experiment. The initial boundary value problems of the linear

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wave propagation theory require a unique solution. Such a solution is independent of choice of the concrete fundamental solution, used in a solving method. The simplest fundamental solution is usually considered the most convenient for practical reasons. The classical Green's function of an unbounded medium satisfying the classical Fermat's principle is often a preferred choice.

The problem becomes more complex when analyzing the full wavefield. Its interference structure needs to be represented as the sum of the source wavefield and the wavefields scattered at the boundaries and medium heterogeneities. The source wavefield is represented by a superposition of fundamental solutions. It is as ambiguous as the fundamental solutions are. A fundamental solution can propagate only inside the actual medium and does not exist out of it. In media of complex geometrical shapes, the fundamental solution may contain artifacts (physically unfeasible wavefields) that propagate along the ray trajectories, partly beyond the boundary of the considered domain. Fundamental solutions that describe observable point source wavefields are considered feasible in this paper. To exclude artifacts from the source wavefield it is necessary to analytically describe the feasible fundamental solution in the domains with arbitrary boundary shapes [1–3].

The problem of describing feasible fundamental solutions was first addressed by Hadamard using the theory of characteristics in 1910. Hadamard described the kinematic properties of these solutions using the generalized Fermat's principle for arbitrary domains [3]. According to this definition, the front of the fundamental solution propagates only along non-classical rays that belong entirely to the domain of consideration. Kinematic properties of the wave front in domains with arbitrary boundaries can be correctly described using the Huygens' principle (see details in Sections 5 and 6 of Chapter 2 in [3]). While the front is inside a considered domain it has a classical shape. Part of the front that intersects a boundary and propagates outside a domain is physically non-feasible and is not further taken into account. The physically feasible part of the front starts to creep into the concave parts of a boundary and propagates into the shadow zones for classical rays. In addition, nonclassical rays propagate inside this domain in the shadow zones for classical rays. Part of these nonclassical rays belongs to a curved boundary of a domain (see Section 5 Chapter 2 in [3]). We thus conclude that physically feasible fundamental solutions depend on the actual shape of the domain.

After Hadamard's work there were numerous attempts to use rigorous or approximate formulations of the initial boundary value problems of mathematical wave theory in order to find physically feasible fundamental solutions. Friedlander gives the detailed rigorous Hadamard's description of the propagation of front of the fundamental solution for concave boundaries [3]. Although the generalized Fermat's principle, as introduced by Hadamard, states that it is necessary to exclude the nonphysical components of the fundamental solution, it does not provide a solution for how to obtain the feasible fundamental solution.

The problem of obtaining the feasible fundamental solution first appears in the work of Kirchhoff in 1881, where a heuristic principle of absolute absorption was proposed [4,5]. Let us consider this principle with the example of a homogeneous acoustic domain. In a convex domain this principle is not applicable as radiation propagates from any point source to any boundary point along the ray. Therefore, in such a domain, a point source wavefield can be computed at any point of a boundary. In a concave–convex domain this principle should be applied because radiation propagates from a point source along rays only to points of the 'illuminated' parts of the boundary. Radiation does not propagate to points of the 'shadowed' parts of the boundary because the ray is intercepted by a 'shadowing' convex part of the boundary. In such a situation Kirchhoff suggested to take into account 'absolute absorption' at 'shadowed' concave parts of the boundary by the vanishing a wavefield at points of the 'shadowed' parts of the boundary.

Kirchhoff attempted to justify this principle [4,5]. He obtained an approximate description of the fundamental solution for a half-plane slit in a homogeneous medium. Several papers show that direct application of Kirchhoff's principle leads to the fundamental solutions containing unadmissible singularities in the vicinity of the edge bounding the illuminated part of the boundary [4,5].

For practical reasons, the contemporary research focused on the problems of the scattering of plane, cylindrical and spherical waves in homogeneous media with simple boundaries. Some of the approaches used are: the method of variables' separation; the method of spectral decomposition; the theory of multiple diffraction based on the locality principle [6], allowing addition of diffraction in source wavefield in shadow zones; the theory of edge and tip waves [7,8]; and the hybrid (numerical-asymptotic) boundary integral method [9]. Rigorous methods are applicable to describe diffraction at wedge-shaped boundaries [3,6,7,9]. A combination of the spectral decomposition method and locality property is applied to diffraction at polygons and polyhedrons [6,9]. Various approximate methods of calculation of the fundamental solution are applicable to diffraction at concave boundaries (circular, parabolic or hyperbolic cylinders) of open domains [3,5]. All the proposed approaches satisfy the generalized Fermat's principle inside geometrical shadow zones.

The exact analytical solution of all rigorous diffraction problems takes into account the geometrical shadow zones for the direct wavefield. As an example, we consider a problem of an impulse diffraction at a wedge with perfect boundary conditions. The detailed description of the solution of the problem, Green's function, is represented by formula (5.2.10) in [3] (see Fig. 5.2). Green's function is represented by the sum of the direct wavefield (5.4.6) and the reflected wavefield which is out of the scope of this paper. The direct wavefield is composed of the direct wave with its shadow zone and the diffracted wave, smoothing a discontinuity in amplitude at the shadow boundary. Time arrival of the direct wavefield satisfies the generalized Fermat's principle as front of the diffracted wave in the shadow zone retards with respect to the standard Fermat's principle. The direct wavefield can be considered as the feasible fundamental solution in any shadowed domain.

Revival of interest in the theory of feasible fundamental solutions in media with complex boundaries is stimulated by the introduction of an analytical solution of the initial boundary value problem for layered medium with curved interfaces