Wave Motion 52 (2015) 33–53

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Three-dimensional modeling of elastic guided waves excited by arbitrary sources in viscoelastic multilayered plates

Fabien Treyssède

LUNAM Université, IFSTTAR, Route de Bouaye, 44344 Bouguenais, France

HIGHLIGHTS

- A modal solution is derived for the 3D modeling of Lamb and SH waves.
- Sources can be of arbitrary shape.
- The solution remains applicable to viscoelastic plates and in the near field.
- Formula are presented to calculate point source excitabilities from lines sources.
- The theory is validated by literature solutions and 2D convolution results.

ARTICLE INFO

Article history:
Received 17 January 2014
Received in revised form 2 June 2014
Accepted 24 August 2014
Available online 4 September 2014

Keywords:
Waveguide
Plate
Excitation
Three-dimensional modeling
Viscoelastic
Excitability

ABSTRACT

This paper provides a modal solution for the three-dimensional modeling of Lamb and SH waves excited by sources of arbitrary shape. This solution is applicable to elastic and viscoelastic plates, in the far-field as well as in the near-field regions, under the assumption of transverse isotropy about the thickness direction. The theoretical developments are conducted based on a semi-analytical finite element formulation. This formulation yields a one-dimensional modal problem, fast from a computational point of view, and allows to readily handle heterogeneous materials having depth-varying properties (multilayered, piecewise or continuously varying, functionally graded). The modal solution is shown to be expressed in terms of Hankel functions of multiple order thanks to a proper application of inverse transforms and Cauchy residue calculus. The link between the proposed formulation and a fully analytical approach is discussed. The solution of this paper is then successfully compared to literature results and degenerates to the point source case. Formulas are presented to calculate point source excitabilities from lines sources. These formulas remain valid for non-propagating modes, viscoelastic materials and account for the near-field contribution. Finally, the example of a viscoelastic bilayer waveguide excited by a rectangular source is considered in order to check the theoretical results.

1. Introduction

Lamb waves are of great interest for the non-destructive testing and the health monitoring of plate-like structures. Such waves are dispersive and multimodal, which complicates their practical use. Dispersion curves of phase and group velocities as functions of frequency are useful to identify modes that propagate in a frequency range with low dispersion and low attenuation [1]. These curves represent modal properties obtained regardless excitation. For a practical inspection system, it is also essential to determine and control the amplitudes of each guided modes excited by a given source. This information typically allows to optimize the type and location of sensors to be used.

E-mail address: fabien.treyssede@ifsttar.fr.

http://dx.doi.org/10.1016/j.wavemoti.2014.08.007

The 2D modeling of Lamb wave excitation and propagation has been extensively studied. Two approaches can mainly be distinguished in order to calculate the response of waveguides under excitation. A first approach consists in using integral transform techniques [2–7]. With this method, the response is obtained by contour integration in the complex plane and residue calculus, or numerical integration, of the analytical solution expressed in the wavenumber domain.

An alternative approach is based on modal analysis, which consists in expanding the excited field as a sum of guided modes [8–10]. The contribution of each mode is obtained from an orthogonality relation between eigenmodes. This second method appears to be more suitable to achieve a better interpretation and optimization of signals, as it directly provides the contribution of each mode as a function of the excitation. Additionally, this method has allowed to introduce the useful concept of modal excitability [11–14]. For a given frequency, the excitability of a particular mode can be defined as the ratio of the displacement of that mode to a point force applied in a given direction.

However, the plane-strain assumption inherently used for 2D modeling implies that guided waves are not geometrically attenuated as they propagate and that any source actually extends infinitely along the out-of-plane direction (a 2D point force hence corresponds to an out-of-plane line source). A 3D formulation is required in order to account for a finite length excitation, typically generated by transducers.

The 3D modeling of Lamb wave propagation generated by finite sources is more complex. Based on contour integration and residue calculus, point source solutions can be found for isotropic plates [15–19]. Mainly based on numerical integration, further works can be found for quasi-isotropic [20], multilayered isotropic [21] or anisotropic [22–24] plates. However, numerical integration techniques usually require a large amount of computation time to evaluate the double integral of the spatial inverse Fourier transform, which has extremely irregular integrands [22,23].

With modal techniques, a direct way of expressing 3D wave fields in terms of modal expansions has been formulated by Achenbach [9] for an isotropic plate. This novel approach uses reciprocal identities as well as the concept of carrier waves [25] and leads to analytical solutions both for normal and tangential point loads [26]. The advantage of a fully modal technique is to replace the double integral of the inverse Fourier transform with a modal expansion, which is much more efficient from a computational point of view. Based on the work of Achenbach, Wilcox [27] has derived asymptotic far-field 3D modal excitabilities. Under the assumption that no material damping is present, Velichko and Wilcox [13] have further extended these results to generally anisotropic multilayered media. Based on Refs. [2,18], Moulin et al. [28] have proposed far-field modal solutions for an isotropic plate subjected to a normal surface load of rectangular shape. More generally, problems involving arbitrary sources can be treated by two-dimensional convolution of the point source solution, but this can be costly from a computational point of view. Further improvements are possible.

The purpose of this paper is to provide a 3D modal solution for Lamb and SH waves generalized to sources of arbitrary shape. This solution is restricted to transversely isotropic problems with symmetry axis normal to the plate surface (quasi-isotropy), so that the modal features of the plate do not depend on the propagation angle. This key property allows to achieve fully modal solutions (without integral), expanded as double sums over normal modes and Fourier coefficients of the source. This paper generalizes previous 3D modal solutions usually restricted to point sources [26,27,13]. Furthermore, the proposed modal solution is shown to be applicable to viscoelastic solids as well as in the near field region. It should be mentioned that, while the integral transform approach still applies with complex poles, and thereby to lossy waveguides [29,24,30], the validity of modal techniques with complex modes might be unclear. Complex modes typically occur with viscoelastic materials or in near-field calculations, involving evanescent or inhomogeneous modes. In case of 2D plate modeling, it has been recently shown in Ref. [14] that complex modes can be handled with modal expansion techniques thanks to the use of Auld's real biorthogonality relation, instead of Auld's complex relation [8] (the latter only holds for real wavenumbers, i.e. propagating modes in lossless waveguides).

The theoretical developments of this paper are mainly based on a so-called semi-analytical finite element (SAFE) method. Although approximate by nature, such a numerical method allows to readily handle heterogeneous materials having depth-varying properties (multilayered, piecewise or continuously varying). The SAFE modal approach has been essentially developed for studying 3D cylindrical waveguides of arbitrary cross-section, viscoelastic or not (see for instance Refs. [31–35]), and 2D anisotropic multilayered plates subjected to line loads [36,37]. For plate structures, the SAFE method restricts the finite element (FE) discretization to only one dimension (along the depth) and is thus fast from a computational point of view. Wave modes can be solved from a matrix eigensystem using standard eigensolvers, which avoids the use of complex root finding algorithms required with fully analytical approaches [38,39]. A one-dimensional SAFE approach has been specifically proposed by Bai et al. [40] for computing the 3D response of layered isotropic plates. These authors yet restricted their calculation to Green's solution (point source). Besides, the solution was not expressed in terms of Hankel functions, which may limit its practical use. In the present paper, the source is of arbitrary shape and the wavenumber domain of integration is chosen differently, leading to Hankel type solutions.

This paper is organized as follows. Section 2 describes the SAFE formulation required for the 3D modeling of elastic waves. In this formulation, the displacement field retains its three components. Section 3 gives the modal solution. The response is first derived in the wavenumber domain. The response in the space domain is then obtained from the application of the Cauchy residue theorem. It is shown that the solution is a double sum on Lamb modes and on the Fourier coefficients of the excitation, involving Hankel functions of multiple order. The link between the SAFE solution and a fully analytical approach is also established. In Section 4, the modal solution is validated with literature results. Formulas are presented to calculate point source excitabilities from lines sources. A discussion on orthogonality relations is provided to highlight the closed link existing between the SAFE biorthogonality relation, the real biorthogonality relation of Auld [8] and the relation