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Vibration analysis of a multi-span rotating ring with ray tracing method

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HIGHLIGHTS

- The ray tracing method is more efficient and faster approach in the dynamic modeling of rotating structures.
- The ray tracing method is able to significantly reduce the computational complexity of the characteristic equation and the impulse response.
- It can be extended to calculate the complex rotating structures.

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ABSTRACT

A ray tracing method is applied to analyze both the free and forced responses in a rotating multi-span section ring. In this paper, element coordinates are established and coupled by dispersion and transmission matrices. Structure vibration displacements are expressed in a wave form with a combination of propagation, fast-attenuating and near-field waves. Meanwhile, an exciting force is considered as a point discontinues with different elements on both sides. The wave reflection and transmission matrices are introduced through coupling different elements by applying wave transmission coefficients and transfer matrices. For numeric computation, the reflection and transmission matrices are assembled, independent waveguide elements are integrated and the responses of rotating rings with non-uniform section area are derived. The structure modeling and a numeric computation with corresponding solutions illustrate the validity of the presented approach. The investigation result also shows that the presented approach can be extended to compute accurately on the dynamic characteristics of a rotating complex structure (high speed bearing cage).

1. Introduction

Most of the engineering constructions are comprised of elements, and these elements can be modeled as waveguide structures such as thin wall, straight beam and bending beam. An efficient computing method relating the geometric and size parameters to predict the structure's dynamic characteristics will provide an effective way to aid structure design and mechanism optimization.

Love [1] presented primitive studies about the wave motion in a curved beam. In his book, an equivalent radius of curvatures and an in-extensional neutral axis were both assumed. Not until the phase-closed principle had been proved by Mead [2], the wave propagation method was systematically used in the vibration analysis of various structures, including beams [3,4], shafts [5] and so on. Kang [6] considered the free vibration of circular curved beams as multiple spans with

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discontinuities and general boundaries. Chouvion [7,8] applied the Ray Tracing Method (RTM) to study the in-plane and out-of-plane vibration of static ring-beam structural point and largely reduced the modeling complexity.

Another important issue is the vibration of rotating rings which has application background in a solid cage of a high speed roller bearing. Mallik and Mead [9] investigated the natural frequencies and mode shapes of static rings, as one of the earliest utilization of wave analysis techniques. Endo [10] studied experimentally the flexural vibration characteristics of a rotating ring. Bickford [11] presented a derivation of the governing equation for the motion of rotating rings in plane and neglected transverse effects. Williams [12] determined the forced response of in-plane rotating rings by the variable separation. Huang [13] derived general equations of motion with effects of Coriolis acceleration and expanded the solution to the forced response in the usual manner. Kim [14] considered the non-linearity and coupling of the in-plane and out-of-plane displacements, and studied the natural frequency of a rotating ring by a perturbation method. Therefore, a systematic research is still needed to be done for the problem raised in a high speed roller bearing cage.

An approach that also considers the scattering and propagation of waves is the RTM. It was employed in the near-field wave component neglected to predict the response of coupled beams subjected to flexural vibrations only. It was later extended to static planar circular curved beam structures including the effects of attenuating components [7] and to the free vibration analysis of complex planar ring-beam structural systems, including cyclic symmetry. This matrix-based method offers a compact and systematic methodology that allows complex structures, such as multi-span beams, trusses and aircraft panels with periodic supports to be analyzed. The main purpose of this paper is to establish a model of a rotating ring for computer implementation which can be used for the free and forced vibration response analyses, and extend the RTM to solve the dynamic problem of a rotating arraying structure and a rotating couple structure combining a ring and beam elements.

The manuscript is organized as follows. Section 2 presents the ray tracing approach to calculate the free and forced vibration responses of waveguide structures. Section 3 provides the matrix method for computer implementation and the calculation of the transmission coefficients for the case of N connected elements. Section 4 applies the presented method to illustrate example of the solid cage in bearing.

2. Ray tracing method

2.1. Displacement definition

Euler–Bernoulli beam theory is used in this paper while shear deformation and rotary inertia are not considered. Based on this theory, the fundamental governing equations are derived from Ref. [15] and are used extensively in later computations.

A wave traveling approach is utilized in this paper to define the displacement of the element in the curved beam. This approach is also the corner stone for building the ray tracing method. From the equation of motion, the displacement of the curved beam can be expressed as a superposition of several pair positive traveling waves and negative traveling waves. For the curved beam element, the in-plane radial displacement is defined as w, the flexural displacement along the centerline is defined as u. The displacement can be expressed as follows,

$$w(s) = C^{+}_{w1}e^{i\gamma_1 s} + C^{+}_{w2}e^{i\gamma_2 s} + C^{+}_{w3}e^{i\gamma_3 s} + C^{-}_{w4}e^{i\gamma_4 (s-L)} + C^{-}_{w5}e^{i\gamma_5 (s-L)} + C^{-}_{w6}e^{i\gamma_6 (s-L)}, \tag{1a}$$

$$u(s) = \alpha_1 C^{+}_{w1}e^{i\gamma_1 s} + \alpha_2 C^{+}_{w2}e^{i\gamma_2 s} + \alpha_3 C^{+}_{w3}e^{i\gamma_3 s} + \alpha_4 C^{-}_{w4}e^{i\gamma_4 (s-L)} + \alpha_5 C^{-}_{w5}e^{i\gamma_5 (s-L)} + \alpha_6 C^{-}_{w6}e^{i\gamma_6 (s-L)}, \tag{1b}$$

where $C^{\pm}_{wm}$ $(m = 1, 2, 3, 4, 5, 6)$ indicates the wave amplitude of the traveling wave in the positive and negative direction respectively. α_m $(m = 1, 2, 3, 4, 5, 6)$ is defined as the ratio of the radial to the tangential amplitudes of wave traveling in the same direction along a curved beam. These coefficients can be derived from the motion equation [4]. γ_m $(m = 1, 2, 3, 4, 5, 6)$ is the complex wavenumber associated with the waves of amplitude $C^{\pm}_{wm}$. For more details of wave characters in a rotating curved element, the reader may move to Ref. [15].

2.2. Ray tracing

As shown in Fig. 1, wave incidents at the boundary of each element and travels in the positive and negative directions inside each element. Assume that the whole waveguide is made up of N elements and the initial wave amplitude vector is expressed as the following vector:

$$\mathbf{c}_0 = \begin{bmatrix} \mathbf{c}^{+}_{1initial} & \mathbf{c}^{-}_{1initial} & \cdots & \mathbf{c}^{+}_{Ninitial} & \mathbf{c}^{-}_{Ninitial} \end{bmatrix}^T. \tag{2}$$

Element length and wavenumber can affect the propagation and their effects can be defined as a decaying term, expressed as a dispersion matrix $\mathbf{D}$. All terms in $\mathbf{D}$'s diagonal are in the form of $D_{pp} = e^{i\gamma_p L_q}$ while the rest terms are zero, where γ_p is the wavenumber associated with wave p and L_q is the element length through which it travels (θ_q is used in a curved beam). When the wave propagates into a discontinuity, the wave will transmit partly into the neighbor element and reflect partly into the original element. For example, the incident flexural wave would trigger a combination of waves containing flexural wave, decaying wave and longitudinal wave. In this paper, complex coefficients are utilized to express the different nature of these waves. Considering the equilibrium and continuity expressions at the joint of two elements, those coefficients can be solved. The scattering in the overall structure wave can be expressed by a transmission matrix $\mathbf{T}$ where $c_p = \mathbf{T}_{pq}c_q$ and $\mathbf{T}_{pq}$ is the transmission coefficient from a wave of amplitude c_q to a wave of amplitude c_p.