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A finite difference method for elastic wave scattering by a planar crack with contacting faces

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HIGHLIGHTS

- A finite difference (FD) method for elastic wave scattering by cracks was proposed.
- Stick and frictionless contact conditions may be implemented in the FD method.
- Higher-harmonic generations due to crack face contact were simulated numerically.
- Scattered shear waves can be a rich source of information on the contact condition.
- The FD method is useful for simulations of ultrasonic testing of closed cracks.

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ABSTRACT

This paper presents a finite difference time-domain technique for 2D problems of elastic wave scattering by cracks with interacting faces. The proposed technique introduces cracks into the finite difference model using a set of split computational nodes. The split-node pair is bound together when the crack is closed while the nodes move freely when open, thereby a unilateral contact condition is considered. The development of the open/close status is determined by solving the equation of motion so as to yield a non-negative crack opening displacement. To check validity of the proposed scheme, 1D and 2D scattering problems for which exact solutions are known are solved numerically. The 1D problem demonstrates accuracy and stability of the scheme in the presence of the crack-face interaction. The 2D problem, in which the crack-face interaction is not considered, shows that the proposed scheme can properly reproduce the stress singularity at the tip of the crack. Finally, scattered fields from cracks with interacting faces are investigated assuming a stick and a frictionless contact conditions. In particular, the directivity and higher-harmonics are investigated in conjunction with the pre-stress since those are the basic information required for a successful ultrasonic testing of closed cracks.

1. Introduction

When a solid–solid interface in imperfect contact is subjected to an incident wave, scattered waves with both excitation (fundamental) and higher-harmonic frequencies are generated due to a nonlinear interaction of the interface [1]. In the field of ultrasonic nondestructive testing (NDT), this nonlinear phenomenon is called the contact acoustic nonlinearity (CAN), and has been studied theoretically and experimentally. Buck et al. [1] measured the second harmonic wave generated at

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an aluminum–aluminum interface as a function of contact pressure. They also observed a second harmonic generation due to an interaction between surface waves and micro fatigue cracks in an aluminum alloy. Cantrell et al. [2] showed that the second harmonic is excited by fatigue-induced micro cracks in an aluminum alloy in response to longitudinal ultrasonic waves. Moreover, they found that a nonlinearity parameter defined as a ratio of the 2nd to the 1st harmonic amplitude grows monotonically with an increase in fatigue cycles. Kawashima et al. [3] demonstrated that a CAN-based ultrasonic imaging can be a powerful tool in detecting and characterizing defects which are not visible to classical linear ultrasound. For example, they showed that higher-harmonics can detect a fiber/matrix debonding, which is considered as a fracture process zone of a matrix cracking in a CFRP laminate. They also showed that micro voids at a diffusion bond interface can be visualized only in higher-harmonic images because the bond interface strongly reflects the waves of the excitation frequency. Feasibility of the higher harmonic imaging has been demonstrated at a larger scale. Yun et al. [4] used aluminum blocks with polished, partly indented but otherwise planar surfaces to form a partially contacted aluminum–aluminum interface. They used a synthetic aperture focusing technique to visualize the interface and showed that the contacted portion of the interface appears only in the higher-harmonic images.

Several models have been proposed to explain the mechanism of harmonic generation due to CAN. Richardson [5] used a one-dimensional model of semi-infinite elastic bars to investigate the harmonic generation and its excitation efficiency. The bars are assumed to be in unilateral contact where only a compressional stress is transmitted, while tensile motion at the interface creates a gap with vanishing traction. Imperfect interfaces have also been modeled by an interfacial stiffness (e.g., [6,7] and the references therein) which requires continuity in the stress while allowing the displacement to be discontinuous. Nagy [7] reviewed the theoretical and experimental results relevant to this problem to conclude that the transverse reflection coefficients at imperfect interfaces can be used to distinguish a kissing bond from a partial bond interface. Margetan et al. [6] used the interfacial stiffness concept in conjunction with a quasi-static model and successfully reproduced a measured, frequency dependent reflection coefficient in a low-frequency regime. It is known however that the linear interfacial stiffness does not excite higher harmonics upon reflection and transmission of elastic waves. To make the interfacial stiffness concept more complete, Biwa et al. [8] introduced a quadratic form interfacial stiffness having pressure dependent linear and second order (nonlinear) stiffness constants. Based on the experimental findings given in foregoing researches such as [9], they assumed that the linear stiffness is related to the ambient pressure by a power law and derived an expression for the second-order stiffness. The 1st and 2nd harmonic amplitudes are obtained thereafter by solving the governing, 1D, nonlinear wave equation by a perturbation technique. Biwa's 1D model was extended further by Nam et al. [10] to a planar infinite interface subjected to an obliquely incident plane wave. Their model was validated by comparing measured and theoretical 2nd harmonic amplitudes.

Because of the nonlinearity of the problem, it is rather difficult to obtain exact elastodynamic solutions for higher harmonic generations except in a very simple setting (e.g., 1D medium with a gap subjected to a monochromatic incident wave). It is hence necessary to use a numerical method to solve more general scattering problems involving interacting interfaces. One of the numerical methods which has been used for this purpose is boundary element method (BEM). Mendelsohn and Doong [11] formulated the scattering problem as a time-domain boundary integral equation for 2D, in- and anti-plane waves. Cracks were modeled by mutually non-penetrating faces which support a Coulomb friction force. They solved the integral equation for the anti-plane wave numerically, and obtained scattered fields from a surface breaking crack. The in-plane problem was solved by Hirose [12] using a time-domain BEM based on a hyper-singular integral equation. Generation of higher-harmonics in the presence of pre-stress was clearly demonstrated. Hirose and Achenbach [13] applied their boundary element technique to a 3D problem in which a penny-shaped crack with contacting faces is excited by a normally incident plane L-wave. Some authors used FEM to solve similar scattering problems. Delrue and Abeele [14] simulated dynamic response of a circular delamination in a composite material. They modeled the interaction between the delaminated surfaces by a multi-linear interfacial stiffness with a viscous damper. A commercial finite element platform was used for their numerical analysis, and harmonic generations were investigated. Blanloeuil [15] considered a scattering by a crack whose faces are in unilateral contact bearing a Coulomb friction force. The numerical solver used was a finite element based on a penalty method although the numerical implementations were scarcely detailed. Harmonic generations by an obliquely incident P- and SV-wave of various intensities were simulated trying to gain insights into CAN.

In this study, a finite-difference time domain (FDTD) method is proposed for analyses of the elastodynamic scattering by a crack with interacting faces. The finite difference (FD) method has not been applied to stress analyses of cracked media as often as FEM and BEM because it is widely believed that the stress singularity at the crack tip cannot be reproduced accurately unless highly elaborated scheme is used [16]. However, it has been shown by Masserey and Mazza [17] that a FD with staggered grids and split nodes on the crack faces can cleverly reproduce the singular stress field. Motivated by their work, the FDTD for linear elastodynamics is used in this study with the split computational nodes in order to develop a numerical technique useful for the analyses of CAN involving either cracks or interfaces. The FDTD based on the 1st order PDE system is used in this study because Masserey's approach based on the 2nd order system suffers some difficulties in developing an explicit scheme when crack-faces interact.

The major disadvantage of FD methods is inflexibility in the model geometry, which can be remedied to some extent by refining the mesh. On the other hand, well-studied stability properties, straightforward numerical implementations, computational efficiency, and the scalability are great advantages. Those advantages are highlighted if a time domain BEM was employed as a numerical solver. It is known that the stability of the time-domain BEM is problem dependent and in general not good [18]. For example, exterior Neumann and Dirichlet problems have different stability properties [19].