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Wave propagation in graded rings with rectangular cross-sections

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ABSTRACT

In this paper, a double orthogonal polynomial series method is proposed to investigate the guided wave propagation in a two-dimensional (2-D) structure, namely, a FGM ring with a rectangular cross-section. Two kinds of graded rings are considered: material gradient directions being in the radial direction and in the axial direction respectively. Numerical comparison with available reference results for a straightly homogeneous rectangular bar illustrates the validity of the proposed method. The dispersion curves and displacement distributions of various FGM rings, which have different radius to thickness ratios, different material gradient directions and different thickness to height ratios, are calculated to reveal the guided wave characteristics.

1. Introduction

With the gradually changing material properties, functionally graded materials (FGM) have been tailored for many worse working environments, such as high temperature difference and strong corrosive environments. For the purpose of the guided ultrasonic non-destructive testing and evaluation (NDT&E), guided wave propagation in FGM structures has received considerable research efforts. As early as in 1991, Liu et al. [1] investigated wave propagation in FGM plates. Liu and his co-workers proposed homogeneous and inhomogeneous (linear and quadratic) finite layer element methods to investigate FGM [1,2] and FGPM (functionally graded piezoelectric material) plates [3], FGM [4] and FGPM [5] hollow cylinders. A graded spectral element method was developed by Chakraborty and Gopalakrishnan [6,7] to study wave propagation in FGM beams. Wang and Rokhlin [8] used recursive geometrical integrators method to investigate wave propagation in graded multilayered elastic composites. By means of reverberation matrix method, Chen et al. [9] have calculated the dispersion curves of FGM plates. With the Galerkin finite element and Newmark methods, Hosseini [10] obtained the axisymmetrical dynamic solution for an isotropic layered FGM thermoelastic hollow cylinder without energy dissipation. All the above mentioned research works divide the FGM structures into many homogeneous or inhomogeneous layers. Some other research works treated FGM structures as continuously graded media. Sun and Luo [11–13] used higher-order shear

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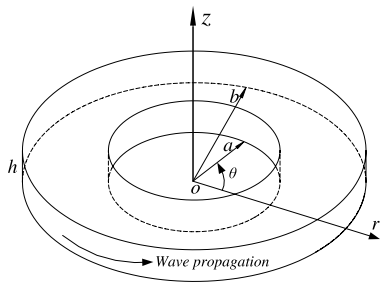

Fig. 1. Schematic diagram of a ring with rectangular cross section.

deformation theory to investigate the elastic wave and thermoelastic problem in FGM plates. Xue and Pan [14] analytically studied the axisymmetrical longitudinal wave in a functionally graded magneto–electro–elastic rod with material properties being assumed to vary exponentially along the rod direction. Legendre polynomial series method was developed by Lefebvre et al. to investigate wave propagation in FGPM plates [15] and FGM hollow cylinders [16]. The polynomial series method has a special feature in the sense that it incorporates the boundary conditions into the equations of motion. So the boundary conditions are automatically accounted for by assuming position-dependent material parameters. The equations of motion are then converted into a matrix eigenvalue problem through the expansion of the independent mechanical variables into appropriate series of orthonormal functions which makes the semi-variational determination of the frequencies and the associated modes possible. By using the polynomial series method, Yu et al. investigated the guided wave characteristics in FGM spherical plates [17] and magneto–electro–elastic FGM structures [18,19]. Yu et al. also extended this method to thermoelastic [20] and viscoelastic [21] FGM plates to analyze the wave attenuation resulting from the thermal effect and the viscosity.

According to the above simple review, guided waves in various FGM waveguides have been investigated, but these waveguides are all one-dimensional structures, i.e., structures having a finite dimension in only one direction, such as horizontally infinite flat plates and axially infinite hollow cylinders. To the authors' knowledge, guided wave propagation in two-dimensional (2D) FGM waveguides has not yet been reported in literature. But 2D waveguides are widely used in engineering structures, such as straightly bars, rings and so on. This paper proposed a double orthogonal polynomial series method to investigate the guided wave propagation in FGM rings with rectangular cross-sections. Two kinds of graded rings with different gradient directions are considered: material gradient directions being in the radial direction and in the axial direction respectively. Dispersion curves and displacement profiles of various FGM rectangular rings are presented. In this paper, traction-free boundary conditions are assumed.

2. Mathematics and formulation of the problem

We consider an orthotropic ring with rectangular cross-section, as shown in Fig. 1. In the cylindrical coordinate system (r, θ, z), a, b are the inner and outer radii. d is its thickness in the r direction and h is its height in the z direction. The radius to thickness ratio is defined as $\eta = b/(b-a)$.

For the wave propagation considered in this paper, the body forces are assumed to be zero. Thus, the dynamic equation for the ring is governed by

$$\begin{aligned}
&\frac{\partial T_{rr}}{\partial r} + \frac{1}{r}\frac{\partial T_{r\theta}}{\partial \theta} + \frac{\partial T_{rz}}{\partial z} + \frac{T_{rr} - T_{\theta\theta}}{r} = \rho \frac{\partial^2 u_r}{\partial t^2} \\
&\frac{\partial T_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial T_{\theta\theta}}{\partial \theta} + \frac{\partial T_{\theta z}}{\partial z} + \frac{2T_{r\theta}}{r} = \rho \frac{\partial^2 u_\theta}{\partial t^2} \\
&\frac{\partial T_{rz}}{\partial r} + \frac{1}{r}\frac{\partial T_{\theta z}}{\partial \theta} + \frac{\partial T_{zz}}{\partial z} + \frac{T_{rz}}{r} = \rho \frac{\partial^2 u_z}{\partial t^2}
\end{aligned} \tag{1}$$

where T_{ij} and u_i are the stress and elastic displacements, respectively; ρ is the density of the material.

The relationship between the strain and displacement can be expressed as

$$\begin{aligned}
&\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, \qquad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \\
&\varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{\partial u_z}{r\partial \theta}\right), \qquad \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \qquad \varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right),
\end{aligned} \tag{2}$$

where ε_{ij} are the strains.

We introduce the rectangular window function $\mathrm{I}(r, z)$

$$\mathrm{I}(r, z) = \pi(r)\pi(z) = \begin{cases} 1, & a \le r \le b \text{ and } 0 \le z \le h \\ 0, & \text{elsewhere,} \end{cases}$$