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Regular wave integral approach to numerical simulation of radiation and diffraction of surface waves

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HIGHLIGHTS

- Incident wave potential expansion of pulsating Green function is employed.
- Surface-wave radiation and diffraction problem is discretised straightforwardly.
- A regular wave integral technique is developed for the wave body problem.

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ABSTRACT

A regular wave integral method is developed in the discretisation of a linear hydrodynamic problem on radiation and diffraction of surface waves by a floating or submerged body. The velocity potential of the problem is expressed as a solution of a body boundary integral equation involving the pulsating free surface Green function or pulsating free surface sources distributed on the body surface. With the use of a discretisation on the regular wave integral rather than discretisations on the singular wave integral of the Green function as in earlier investigations, the singular wave integral is approximated as an expansion of regular (or nonirregular) wave potentials. Influence coefficients between pulsating free surface source points are computed by the approximate expansion together with Hess–Smith panel integral formulas. Thus the velocity potential solution is evaluated by a boundary element algorithm. The numerical results produced from the proposed method agree well with semi-analytic solution results.

1. Introduction

As a fundamental problem in hydrodynamics, the determination of wave induced forces resulting from radiation and diffraction of surface waves by a floating or a submerged body has been extensively studied. With the use of linear free surface boundary condition and harmonic function property in the fluid domain, the velocity potential of the hydrodynamic problem is represented as a solution of body boundary integral equation involving the pulsating free surface Green function. The equation can be solved numerically by combining panel method and suitable approximation of the pulsating free surface Green function or free surface sources distributed on the body surface (see, for example, Frank [1], Beck [2], Lee and Sclavounos [3], Lee et al. [4], Lee and Newman [5], Damaren [6] and Liu et al. [7]). Instead of using the pulsating free surface Green function, Rankine simple source distribution on body and average water surfaces can be applied to approximate the velocity potential solution (see, for example, Yeung [8], Cao et al. [9] and Mantzaris [10]). For a radial symmetric body undergoing oscillatory wave motion, its analytic solution can be approximated by a single free surface source rather than the

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boundary integral of free surface sources continuously distributed on the body surface. For a heaving or surging hemisphere, the velocity potential solution is decomposed into a free surface source located at the centre of the sphere and a wave-free potential, which is expanded in a series of Legendre polynomials and sinusoidal functions (see Ursell [11], Havelock [12] and Hulme [13]). The unknown source strength and expansion coefficients are determined by the boundary condition of the velocity potential on the hemisphere. This method also applies to the wave resistance problem (see Farell [14]) of a travelling spheroid as a sphere becomes a spheroid under a linear invertible transformation. The development into radiation and diffraction waves around a submerged sphere was given by Wang [15] and further extensions into diffraction waves around a submerged spheroid was obtained by Wu and Eatock Taylor [16] and Chatjigeorgiou [17].

In this paper, we are interested in the three-dimensional body boundary integral equation involving the pulsating free surface Green function, which is expressed as a sum of Rankine source potential G_R, its image source potential $\bar{G}_R$ and a singular wave integral K

$$G = G_R + \bar{G}_R + K.$$

Here K is an integral of wave potential function with the singularity $k_0 = \frac{\omega^2}{g}$ along the wave number domain $k \geq 0$ with respect to ω the corresponding wave frequency and g the gravitational acceleration. To avoid the uncertainty of K around the singular wave number k_0, a regular wave integral K^μ by replacing k_0 with $k_0 + \mathrm{i}\frac{\mu\omega}{g}$ is used to push the singular point off the integral domain $k \geq 0$ and to derive K as (Newman [18] and Havelock [19]) $K = \lim_{\mu\to 0+} K^\mu$. The function G with K replaced by K^μ is known as a dissipative three-dimensional pulsating free surface Green function.

Recently, the author [20] introduced a two-dimensional dissipative translational free surface Green function and provided a straightforward discretisation of the regular wave integral into an expansion of regular (or nonirregular) wave potentials. This discretisation scheme, which is referred to as *regular wave integral method*, is robust in numerical computation of a two-dimensional problem [20] and has been further developed into numerical simulation of three-dimensional translational wave-body motion problems [21,22].

It is the purpose of the present investigation to present the application of the regular wave integral method to the radiation and diffraction problem by using the pulsating free surface Green function and to provide a simple discretisation of the boundary integral equation formulating the wave-body motion problem. The proposed method results are validated in accordance with earlier semi-analytic results [13,15,16].

As a significant difference between the regular wave integral and traditional singular wave integral approaches, the former permits the cancellation of the wave integral singularity giving rise to a straightforward approximation of the integral by a regular wave expansion, while the latter in earlier investigations [18,23] requires attacking the singularity in the singular wave integral.

2. Mathematical formulation

Consider a three-dimensional body undergoing periodic oscillatory motion with a constant frequency ω in a fluid of infinite water depth. Let $Oxyz$ be a body-fixed Cartesian coordinate system so that Oz points upwards and the plane $z = 0$ represents the calm water surface. The velocity potential of the linearised oscillatory fluid motion problem can be represented as

$$\Phi = \mathrm{Re}(\phi e^{-\mathrm{i}\omega t})$$

where ϕ is a stationary potential satisfying the following equations

$$\nabla^2\phi = 0 \quad \text{in the field } z < 0, \tag{1}$$

$$\frac{\partial\phi}{\partial z} - \nu\phi = 0 \quad \text{on } z = 0, \tag{2}$$

$$\sqrt{\mathcal{R}}\left(\frac{\partial}{\partial\mathcal{R}} - \mathrm{i}\nu\right)\phi \to 0 \quad \text{as } \mathcal{R} \to \infty \tag{3}$$

for $\mathcal{R} = \sqrt{x^2 + y^2}$ and the dimensional wave number $\nu = \frac{\omega^2}{g}$ with g the gravitational acceleration.

Let S denote the averaging body surface, $\mathbf{n} = (n_1, n_2, n_3)$ the normal vector field of S pointing into the fluid domain and $\phi_{\mathcal{I}}$ the incident wave potential

$$\phi_{\mathcal{I}} = -\mathrm{i}\frac{ga}{\omega}e^{\nu z - \mathrm{i}\nu x\cos\theta - \mathrm{i}\nu\sin\theta}$$

for an incoming incident wave with the amplitude a at the heading angle θ. The angle $\theta = \pi$ represents head waves and $\theta = 0$ represents following waves.

For the radiation wave problem, the complex velocity potential ϕ satisfies the body boundary condition

$$\mathbf{n}\cdot\nabla\phi = -\mathrm{i}\omega n_\alpha \quad \text{on } S. \tag{4}$$