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On optical Airy beams in integrable and non-integrable systems

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HIGHLIGHTS

- Interaction of Airy beams and solitary waves in local and nonlocal media investigated.
- Integrable and non-integrable equations considered.
- Bäcklund transformation used for integrable equation.
- Interaction in a nematic liquid crystal without and with birefringent walkoff analysed.
- In the absence of walkoff, no momentum exchange between solitary wave and Airy beam.

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ABSTRACT

The interaction of an accelerating Airy beam and a solitary wave is investigated for integrable and non-integrable equations governing nonlinear optical propagation in various media. For the integrable nonlinear Schrödinger equation, by way of a Bäcklund transformation, we show that no momentum exchange takes place, as the only effect of the interaction is to modulate the amplitude of the solitary wave. The latter result also holds for propagation in anisotropic media with birefringent walkoff and nonlocality, as specifically addressed with reference to uniaxial nematic liquid crystals in the absence of beam curvature. When the wavefront curvature characteristic of accelerating Airy beams is accounted for, both asymptotic and numerical solutions show that a small amount of momentum is initially exchanged, with the solitary wave rapidly settling to a state of constant momentum.

1. Introduction

The existence of self-accelerating, shape preserving wavepackets has a long history, having appeared and been rediscovered in different fields at different times. Such wavepackets, given in the linear limit in terms of the Airy function [1], were found by Berry and Balazs [2] as solutions of the (linear) Schrödinger equation and recently rediscovered/revived in optics [3,4], resulting in a resurgence of interest in other areas [5–7]. The existence of self-accelerating Airy beams [8,9] and Airy light-bullets [10] was also demonstrated experimentally. The concept of self-accelerating, shape preserving wavepackets was then shown to be more general, proving both theoretically and experimentally the existence of such beams beyond

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the paraxial approximation under which the Schrödinger equation is valid [11,12]. The Airy function is an example of a general type of solution, termed a similarity solution, of several linear and nonlinear partial differential equations. In the context of integrable equations describing nonlinear wave motion, such as the Korteweg–de Vries (KdV) and nonlinear Schrödinger (NLS) equations, similarity solutions are related to special functions called Painleve transcendents [1,13,14]. The nonlinear counterpart of accelerating Airy beams, given in terms of the second Painleve transcendent, Painleve II, was also rediscovered in the context of nonlinear optics [15,16], both numerically [17] and experimentally [17,18]. The nonlinear Airy (Painleve) beam for the self-focusing NLS equation is similar to the linear Airy beam; in self-defocusing media the Painleve solution only exists for a beam intensity below a threshold. However, in focusing media the nonlinear Painleve beam shows very different behaviour to the linear Airy beam: when its intensity is sufficiently large, the main lobe becomes a soliton, with the rest of the beam remaining of Airy-type [19]. Such a soliton forming behaviour can be expected from the inverse scattering solution of the NLS equation.

This earlier work on accelerating Airy beams dealt with local, self-focusing and de-focusing media for which the optical response has the same width of the beam. However, several optical media exhibit a nonlocal response, wider than the beam and governed by an elliptic equation. Major examples of such nonlocal systems are nematic liquid crystals [20,21] and thermo-optic media [22,23]. Such nonlocal nonlinear media support self-accelerating, shape preserving wavepackets, with form similar to the linear Airy beam [24,25].

Nonlinear media can support both Airy and solitary beams or, simply, solitons. In early studies of solitons of the nonlinear (Kerr response) Schrödinger equation versus solitary waves in general, the character of their collisional interactions was considered a discriminating feature. Hereby, we deal with Airy beams and their interaction with solitary waves in either local or nonlocal self-focusing media with a Kerr or Kerr-like response. In particular, as most optical solitary waves are governed by non-integrable equations, we investigate the relation between integrability and momentum transfer between Airy and solitary beams using a hierarchy of equations: at each step we include more physical features in order to determine the role (and importance) of each. We first study the interaction between an Airy beam and a soliton for the simplest case, the integrable NLS equation. The NLS equation governs optical beam propagation in local media for which the response has the same extension as the beam. Then, we analyse the interaction between Airy beams and solitary waves propagating in nonlocal media, specifically addressing uniaxial nematic liquid crystals [21]. Nevertheless, these equations also apply to nonlinear thermal media [24,25] and certain photorefractive crystals [26]. They also reduce to the NLS equation in one limit [27].

For beams governed by the (local) NLS equation, which is integrable via the inverse scattering transform [28], we find a solution based on the method of averaging [29] for this Airy–soliton interaction using a Bäcklund transformation [30]. The averaging is based on the short wavelength of the Airy beam relative to the soliton half-width. It is then found that there is no momentum exchange between solitary and Airy wavepackets, consistent with the results of a previous numerical study [31]. In nonlocal media, where a Bäcklund transformation cannot be used as the governing equations are not integrable, we extend the basic averaging method behind the Bäcklund transformation and show that a nonlocal response introduces on the Airy beam a rapidly oscillating modulation of the refractive index. In the case of nonlocal nematic liquid crystals (NLC) supporting solitary waves (termed nematicons [21]), we confirm numerically that there is no momentum transfer between an Airy beam and a solitary wave, in agreement with the local limit governed by the NLS equation.

Moreover, since NLC, as well as various photorefractive and parametric crystals, are anisotropic dielectrics with uniaxial properties [32,33], we analyse the role of the walkoff angle between the wavefront normal and the Poynting vector of extraordinarily-polarized beams, accounting for the curvature of accelerating Airy beams. Beam walkoff, in fact, depends on the angle of the wavevector with respect to the optic axis. Our numerical solutions show that an initial transfer of momentum between an Airy beam and a solitary wave rapidly decreases as the latter settles to a constant velocity. Conversely, the asymptotic analysis employed within the paraxial approximation indicates that the role of a curved Airy wavefront decreases with propagation distance. This further implies that, even in anisotropic media, no momentum transfer takes place between interacting Airy beams and solitons for large propagation distances.

2. Airy beam interacting with a spatial soliton: local medium

Let us first analyse the interaction between a soliton of the NLS equation (NLSE) and an Airy beam. The soliton and Airy beams are launched collinear with zero initial momentum at $z = 0$. We then consider the nondimensional NLSE

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u = 0 \tag{1}$$

for the initial condition

$$u(x, 0) = \tilde{\eta}\ \mathrm{sech}\ \tilde{\eta}x + a\,\mathrm{Ai}(x), \tag{2}$$

where Ai is the Airy function [1] and $|a| \ll \tilde{\eta}$, so that the Airy function is a solution of the linearized equation. It was found numerically that in this interaction the Airy beam preserves its identity and passes through the soliton without accelerating it [31], but simply modulating its amplitude and width. By employing an appropriate Bäcklund transformation [30] we will show below that these numerical results have a theoretical basis.

To this end, from the inverse scattering solution of the NLSE [28] we recall that the eigenvalues of the scattering problem with a potential given by (2) are purely imaginary as this initial condition is real. When the Airy beam amplitude $a = 0$, the