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Head-on collision of solitary waves in coupled Korteweg–de Vries systems modeling nonlinear transmission lines

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HIGHLIGHTS

- Head-on collision of solitons in coupled Korteweg–de Vries systems is characterized.
- The extended Poincaré–Lighthill–Kuo method is applied.
- Collision-induced phase shifts are evaluated for each propagation mode.
- The governing wave equation for collision-induced pulses is derived.

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ABSTRACT

The extended Poincaré–Lighthill–Kuo (PLK) method is applied to characterize head-on collisions of solitary waves in a coupled Korteweg–de Vries (KdV) system that has multiple modes supporting solitons. As a simple physically realizable system, we investigate two coupled electrical nonlinear transmission lines (NLTLs), and the proposed method successfully leads to the collision-induced phase shifts and the wave equation that governs the dynamics of the pulses generated by colliding solitary waves.

1. Introduction

Coupled nonlinear wave systems have been investigated for their variety of interacting solitary waves such as the resonant and leapfrogging waves [1–3]. Generically, the weakly dispersive systems are well modeled by the coupled Korteweg–de Vries (KdV) equations. One of the promising methods to obtain the dynamical behavior of coupled KdV systems is the perturbative method. Actually, the perturbative method, which is based on the inverse scattering transform, predicts well the leapfrogging frequency and the emission spectrum of radiation for 1-soliton solutions in KdV systems with appropriate couplings [2].

In this study, the collision of counter-traveling solitons in such a system is investigated. Unfortunately, the collisions of counter-traveling solitons cannot be treated straightforwardly in the framework of such a perturbative approach, because an exact solution corresponding to the counter-traveling solitons is lacking. Instead, we apply the extended

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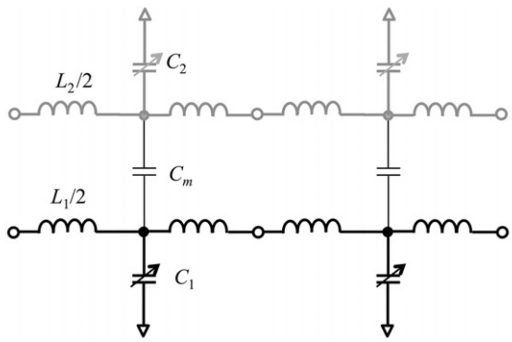

Fig. 1. Two adjacent cells of a coupled NLTL.

Poincaré–Lighthill–Kuo (PLK) method [4]. This leads to explicit expressions for the nonlinearity and dispersion coefficients of the KdV equation in physical dimensions, which governs the small amplitude solitons developed in the investigating physical system and the collision-induced phase shift [5–10]. In coupled systems, the method is expected to describe inter-modal interactions of nonlinear solitary waves quantitatively. It can be applied to any generic coupled nonlinear wave system. Conversely, a system is chosen to apply the method in order to describe the procedure explicitly. As a simple physically realizable example, two electrical nonlinear transmission lines (NLTLs) coupled by capacitors are considered.

When reversely biased, the Schottky diode operates as a capacitor whose capacitance depends on the terminal voltage called the Schottky varactor. An NLTL is defined as a lumped transmission line containing a series inductor and a shunt Schottky varactor in each section. NLTLs are known to simulate the Toda lattice [11]. Moreover, the operation bandwidth of carefully designed Schottky varactors goes beyond 1 THz; therefore, they are employed in ultrafast electronic circuits including the subpicosecond electrical shock generator [12]. Two NLTLs, denoted by lines 1 and 2, are coupled via mutual capacitors. Because of the couplings, there develop two different propagation modes on a linear coupled line called the c and π modes [13]. Each mode has its own velocity and voltage fraction between the lines. In general, a wave travels slower when its wavelength becomes shorter owing to dispersion irrespective of the mode it is carried on [14]; this results in distortions of the baseband pulses having short temporal durations. By introducing Schottky varactors, this distortion can be compensated for by nonlinearity, regardless of the propagation mode. It has been found that the voltage fractions of the c- and π-mode nonlinear pulses are identical with those of the linear c and π modes, respectively.

The extended PLK method leads to explicit expressions for the nonlinearity and dispersive coefficients of the KdV equation, which governs the small amplitude solitons developed in coupled NLTLs and the collision-induced phase shift. In contrast to frequently reported shallow water or plasma systems where the dynamics of a single field variable is required to be solved for every order of the perturbation expansion, we must simultaneously solve the dynamics of two field variables, corresponding to the voltages on lines 1 and 2. By examining the resulting complicated equations, we obtain the KdV equation for the lowest-order field variables for both the c- and π-modes and the phase shift induced by the head-on collision of two similar and dissimilar solitons. Moreover, it is found that the third-order field variables correspond to the pulses generated by colliding solitons, whose governing wave equation is explicitly derived.

Although the procedure is developed in a specific system, it would be equally applicable for other coupled nonlinear wave systems, including two spatially separated plasma systems with electrostatic coupling [15]. The quantitative description of the inter-modal pulse generation might be useful for all such systems.

In Section 2, we discuss the fundamental properties of a coupled NLTL including the structure and propagation characteristics of linear waves. Section 3 is devoted to the head-on collision of two c-mode solitary waves. The collision of two π-mode waves is obtained by the same procedure as in Section 3, whose summarized results are discussed in Section 4. Next, the head-on collision of the c- and π-mode solitary waves is discussed in Section 5. Several numerical calculations are carried out to validate the results obtained by the extended PLK method in Section 6.

2. Coupled NLTLs

Fig. 1 shows the equivalent representation of a coupled NLTL. For the line i ($i = 1, 2$), L_i and C_i represent the series inductor and shunt Schottky varactor of the unit cell, respectively. Moreover, C_m shows the mutual capacitance between lines 1 and 2. Note that there are two ways to connect varactors to line 2. Fig. 1 shows the case where the anodes are connected to line 2. We should equally consider the reversed case, i.e., with the cathodes connected to line 2. Based on this circuit model, the transmission equations of a coupled NLTL are given by [13]

$$L_1 \frac{dI_n}{dt} = V_{n-1} - V_n, \tag{1}$$

$$L_2 \frac{dJ_n}{dt} = W_{n-1} - W_n, \tag{2}$$