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Temporal behavior of laser induced elastic plate resonances

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HIGHLIGHTS

- Local plate resonances are measured with pulsed laser ultrasonic techniques.
- Resonance amplitudes depend on the curvature D of the Lamb mode dispersion law.
- Time decay of zero group velocity Lamb mode resonances scale with $(Dt)^{-0.5}$.
- Time decay of thickness-shear resonances scale with $(Dt)^{-1.5}$.

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ABSTRACT

This paper investigates the dependence on Poisson's ratio of local plate resonances in low attenuating materials. In our experiments, these resonances are generated by a pulse laser source and detected with a heterodyne interferometer measuring surface displacement normal to the plate. The laser impact induces a set of resonances that are dominated by Zero Group Velocity (ZGV) Lamb modes. For some Poisson's ratio, thickness-shear resonances are also detected. These experiments confirm that the temporal decay of ZGV modes follows a $t^{-0.5}$ law and show that the temporal decay of the thickness resonances is much faster. Similar decays are obtained by numerical simulations achieved with a finite difference code. A simple model is proposed to describe the thickness resonances. It predicts that a thickness mode decays as $t^{-1.5}$ for large times and that the resonance amplitude is proportional to $D^{-1.5}$ where D is the curvature of the dispersion curve $\omega(k)$ at $k = 0$. This curvature depends on the order of the mode and on the Poisson's ratio, and it explains why some thickness resonances are well detected while others are not.

1. Introduction

Small amplitude vibrations of an elastic plate are governed by the three-dimensional equations of the linear theory of elasticity. For an infinite homogeneous plate, the frequency $f = \omega/2\pi$ depends on the wavelength $\lambda = 2\pi/k$ in the plane of the plate. When the faces of the plate are free of traction, no energy leakage occurs, then for any real k, the secular equations established by Rayleigh [1] yield an infinite number of real roots in ω. The dispersion curves of these symmetric (S_n) and anti-symmetric (A_n) propagating modes, guided by the plate, are represented by a set of branches in the (ω, k)-plane [2–4]. The complete Lamb mode spectrum depends on material parameters, either expressed by the longitudinal to transverse wave velocity ratio V_L/V_T or by the Poisson's ratio ν [5]. Dispersion curves $\omega(k)$ of high order modes start from the $k = 0$ axis at a finite ordinate ω_c. At these cut-off frequencies $f_c = \omega_c/2\pi$ multiple reflections of longitudinal or shear waves between the top and bottom faces of the plate, give rise to thickness-shear resonances (modes S_{2n} or A_{2m+1}) or to thickness-stretch resonances (modes S_{2m+1} or A_{2n}) at infinite wavelength.

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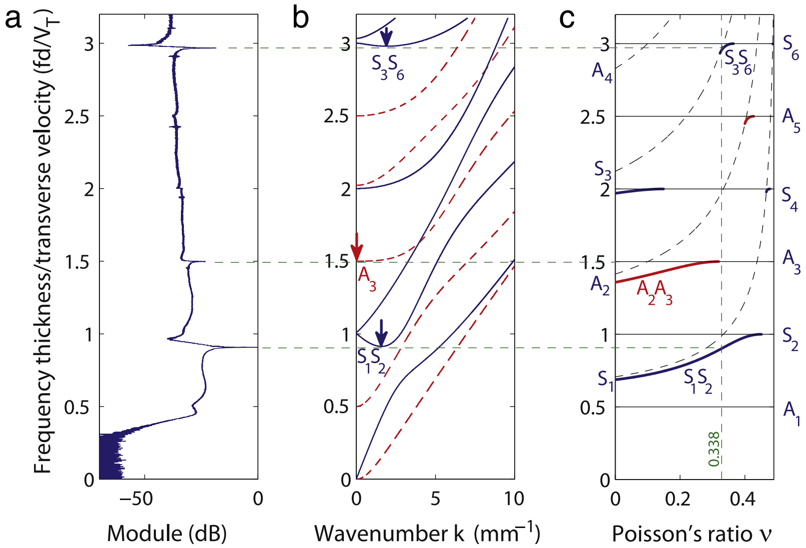

Fig. 1. (a) Spectrum of the normal vibration of a Duralumin plate of thickness $d = 1$ mm generated and detected by laser. Vertical scale: fd/V_T. (b) Normalized dispersion curves for a Duralumin plate with bulk wave velocities $V_L = 6370$ m/s and $V_T = 3150$ m/s ($\nu = 0.338$). (c) Dimensionless cutoff frequencies and minimum frequencies of Lamb modes versus Poisson's ratio (horizontal line: thickness-shear modes, dashed line: thickness-stretch modes, and thick line: ZGV modes).

In many applications, the plate undergoes a local and impulsive excitation. The spectra of the transient waves in an elastic plate has been analyzed by Weaver and Pao [6] and Santosa [7]. When surface stresses are induced by a laser source, the coupling of the thermo-elastic source with elastic waves is a complex problem [8]. The theory and simulation of laser generated waves propagating in plates has been the object of several studies [9,10]. In a recent paper, Laguerre and Tresseyde [11] proposed a method to calculate the excitability of both propagating and non propagating modes. In practice, the energy deposited on the plate by the source of finite dimensions rapidly flows out of the source area except for non propagative modes. Zero group velocity (ZGV) modes were observed experimentally with various techniques: air-coupled transducers [12], impact echo method [13], laser ultrasonics [14,15]. These experiments demonstrate that the local vibration spectrum of a free plate is dominated by the resonance at the minimum frequency of the S_1 Lamb mode. This frequency is slightly lower than the fundamental thickness frequency $f_c = V_L/2d$ and corresponds to the junction of S_1 and S_{2b} branches, where b stands for backward wave [16]. This is why we chose the notation S_1S_2-ZGV resonance [17]. In fact, except for the first three (S_0, A_0 and A_1) Lamb modes, all higher order modes exhibit a minimum frequency for some Poisson's ratio [18,19]. Frequency minima always occur below the cut-off and correspond to wavelengths of the order of the plate thickness. Because of their finite wavelength, ZGV modes dominate the frequency spectrum of the normal surface displacement after a local impact. However, it was observed that some thickness resonances (infinite wavelength) are also detected when the signal to noise ratio is sufficiently high. For example, Fig. 1(a) displays the resonance spectrum measured at the source point (source spot 2.5 mm), on a 1 mm-thick Duralumin plate. The three observed resonances can be identified from the dispersion curves shown in Fig. 1(b). The first and the third ones correspond to the S_1S_2 and the S_3S_6-ZGV modes while the second one is associated to the A_3 thickness-shear mode. Fig. 1(c) presents the dimensionless cutoff frequencies and minimum frequencies of Lamb modes versus the Poisson's ratio: the horizontal lines correspond to thickness-shear modes, the dashed lines to thickness-stretch modes and the thick lines to ZGV modes. For $\nu = 0.338$ (vertical line), only two ZGV resonances exist at normalized frequencies below 3.2. It appears that the A_3 thickness-shear resonance, although 20 dB below the S_1S_2-ZGV resonance, is clearly detected, while for example the A_1 or the A_5 thickness-shear resonances are not observed.

The objective of this paper is to explain these observations using theory, simulation and experiments. It is organized as follows: in Section 2, a simple model is proposed to estimate the temporal decay of ZGV and thickness-shear resonances. The relative amplitudes of thickness resonances, excited and detected by laser techniques, are predicted through simple approximations. Then, Section 3 presents experimental results obtained on Duralumin and fused silica plates. The temporal decay of the different resonances are observed and compared to those obtained with a finite difference code.

2. Analysis of the resonance temporal decay

In order to compare the ZGV and thickness resonance behavior, we propose a simple approach only valid for low attenuation materials. The temporal decay of the S_1S_2-ZGV Lamb mode resonance was studied in Prada et al. [20]. In this paper [Eq. (3)], the normal surface displacement associated to a given Lamb mode was expressed as

$$u(r,t) = \frac{1}{2\pi}\int_0^{+\infty} C_{th}(k)Q(\omega)B(k)J_0(kr)e^{i\omega t}kdk, \quad (1)$$