Wave Motion 50 (2013) 197–209

Contents lists available at SciVerse ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

The damping of viscous gravity waves

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ARTICLE INFO

Article history:
Received 5 June 2012
Received in revised form 11 August 2012
Accepted 17 August 2012
Available online 27 September 2012

Keywords:
Gravity waves
Free-surface flows
Wave damping

ABSTRACT

Using a solution of the linearized Navier–Stokes equations, an approximate formula has been derived for the damping rate of gravity waves in viscous fluids. The proposed solution extends the results found by Lamb (1932) [5] for waves propagating in deep-water conditions for large Reynolds numbers and those derived by Biesel (1949) [14] under more general hypotheses. Specifically, comparisons with the Lamb solution highlight large differences in intermediate and shallow depths and/or for moderate Reynolds numbers while significant discrepancies are observed with the Biesel solution in deep-water conditions. For these reasons, the proposed solution is of great importance for the estimation of the viscous dissipations during the wave motion and represents a useful benchmark for the validation of numerical solvers. With respect to this, the theoretical findings have been compared with numerical simulations obtained by means of a well-known Smoothed Particle Hydrodynamics solver.

1. Introduction

The propagation of gravity waves is an important subject of research because of its applications to various problems and phenomena connected to human works (e.g. coastal and environmental engineering, naval activities, renewable energies, etc.). The greatest part of these phenomena is characterized by high Reynolds numbers ($Re = H_0^*\sqrt{g^*H_0^*}/\nu^*$ where H_0^* is the fluid depth, g^* is the gravity acceleration, ν^* is the kinematic viscosity and stars give dimensional variables) and, consequently, it is often described through the potential theory, that is, assuming that the fluid is inviscid and irrotational. Despite this, dissipations may play an important role when the propagation occurs over long times or when the Reynolds number is not so high. Then, a correct estimate of the dissipative effects turns out to be of fundamental importance for a proper description of the gravity wave evolution.

Many numerical works have been devoted to this subject (e.g. Harlow & Welch [1], Haddon & Riley [2], Raval et al. [3], just to cite a few). Conversely, a further theoretical inspection is still needed, as the largest part of the analytical results just applies to gravity waves in deep-water conditions and with very low viscosity. The first attempts to estimate dissipative effects date back to the works of Basset [4], Lamb [5] (first edition 1879, second edition 1895) and Boussinesq [6]. It is not clear who was the first to find out the attenuation law for gravity waves but, nowadays, this solution is generally referred to Lamb's work. Lamb [5] derived a damping coefficient for the wave amplitude using the linearized Navier–Stokes equations. His approach was based on the assumption that the water depth was infinite and that the viscosity was small. The same result was found by Basset [4] who also tried an extension in finite depths. In this way, he was able to recover the exact linear dispersion relation for the wave celerity but, due to the assumption of low viscosity, he exactly recovered the damping coefficient of Lamb.

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0165-2125/$ – see front matter
doi:10.1016/j.wavemoti.2012.08.008

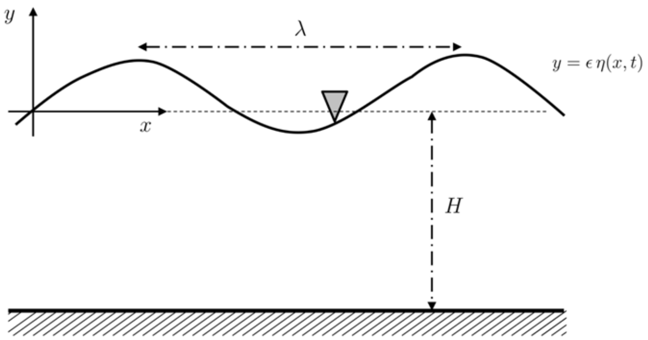

Fig. 1. Sketch of the geometry (dimensionless variables): λ is the wave length, H is the still water depth and $\eta(x, t)$ is the free surface. The dotted line indicates the undisturbed free surface.

Over the years, the same assumptions (that is, deep-water conditions and low viscosity) have been used in several works that confirmed the validity of Lamb's result through different techniques/approaches. For example, Longuet-Higgins [7,8] found Lamb's coefficient by using a boundary layer model and inspecting the equivalence of this model with the theory on weakly-damped waves by Ruvinsky & Freiman [9,10]. Lighthill [11] used the deep-water inviscid solution to approximate the dissipative terms inside the kinetic energy equation for a viscous fluid. In this way, he obtained a damping coefficient for the kinetic energy evolution that can be straightforwardly related to Lamb's one. A more rigorous approach was adopted by Wu et al. [12] to study the fluid motion inside a two-dimensional rectangular tank. Specifically, they solved an initial condition problem for the linearized Navier–Stokes equations using a no-slip condition along the bottom and a free-slip condition along the tank walls. The latter assumption was forced by some theoretical and numerical difficulties in imposing a no-slip condition at the intersection between the free surface and the walls. Despite this, the damping rate predicted by Wu et al. [12] coincides with that predicted by Lamb. More recently, a linear solution of the Navier–Stokes equation has been used by Dias et al. [13] under the hypotheses of infinite depth. They also provided some insight into the action of nonlinearities, showing that the damping rate predicted by Lamb can also be applied to weakly-nonlinear waves.

A significant improvement to Lamb's findings was obtained by Biesel [14] who, thanks to an asymptotic expansion for high Reynolds numbers, derived higher-order terms for the damping rate. Surprisingly, this work is not much refereed to in the current scientific literature. The work of Biesel shows several similarities with the present analysis but, for deep water conditions, it may still lead to an inaccurate prediction of the damping rate.

Incidentally, we highlight that a further subject of research relies on how to include dissipation in models which, usually, do not account for viscous effects. For example, Liu & Orfila [15] derived a system of integro-differential depth-averaged equations that includes dissipative effects due to the bottom boundary layer. Similarly, a viscous-potential formulation was proposed in Dutykh & Dias [16] and further inspected in Dutykh [17] to account for these effects in standard potential theory.

The aim of the present work is to provide a further contribution to the study of viscous gravity waves, that is, waves whose evolution is mainly driven by gravity. Specifically, our analysis is based on a solution of the linearized Navier–Stokes equations over finite depths with a no-slip condition along the bottom. The main result is the derivation of approximate expressions for the wave celerity and the damping rate. Indeed, these also hold true for gravity waves propagating in intermediate- and shallow-water conditions and for Reynolds numbers that are not very large (e.g. $Re \geq 50$). The latter point makes these expressions also suitable for fluids other than water.

For waves propagating in deep-water conditions with large Reynolds numbers, the leading order of the proposed solution reduces to that derived by Lamb. In this case, the higher-order terms provide corrections which are generally small but not negligible. Conversely, when the Reynolds number is not so high and/or the motion takes place in conditions that are far from deep water, the damping rate displays significant differences with respect to that predicted by Lamb. As a consequence, the proposed solution may be important for the estimation of the viscous dissipations during the wave motion. Further, it may be properly used as a benchmark for the validation of numerical solvers (see, for example, Carrica et al. [18]). With respect to this latter topic, we propose some applications in the final part of the present work where the analytical results are compared with the numerical outputs of a Smoothed Particle Hydrodynamics scheme (SPH hereinafter).

The paper is organized as follows: the analytical solution and the damping coefficient are described in Section 2 while Section 3 contains applications and comparisons with the Lamb's and Biesel's solutions.

2. Approximate analytical solution for viscous gravity waves

Let us consider the evolution of viscous gravity waves propagating over finite depths. Fig. 1 displays a sketch of the problem and of the Cartesian frame of reference, whose origin is at the undisturbed free surface with the y-axis pointing upward. Hereinafter, the unstarred variables denote dimensionless quantities. We introduce the nonlinearity parameter $\epsilon = 2A_0^*/H_0^*$ where H_0^* is the reference depth in still-water conditions and A_0^* is the wave amplitude. Accordingly, we use