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On the dispersion of wedge acoustic waves

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ABSTRACT

Acoustic waves guided at the apex of an ideal infinite elastic wedge are non-dispersive. Weak dispersion arises due to a variety of factors. Three of them are investigated in detail: (i) Coating of one or both of the two surfaces of the infinite wedge, (ii) truncating the wedge at its apex or replacing the tip of the wedge by a different material, (iii) slight modification of the material constants of the wedge material in an extended spatial region of the wedge near its tip. These three cases have been analysed within the perturbation theory, and the third case, in addition, with the help of semi-analytic finite element calculations. The dependence of the frequency on wavelength is derived for all three cases, and quantitative results are presented for the dispersion laws of example systems.

1. Introduction

Acoustic waves guided by the apex of an ideal homogeneous elastic wedge have been discovered in the early nineteen seventies [1,2] and have recently found renewed interest in connection with potential applications in non-destructive testing [3,4], ultrasonic motors [5,6], aquatic propulsion [7–9] and acoustic streaming [10]. They are also expected to give rise to interesting nonlinear effects [11–14]. Within the elastic continuum theory, an ideal homogeneous wedge does not define any length scale. Consequently, acoustic waves propagating in such systems and having wavelengths much larger than the interatomic spacing are non-dispersive. This fact causes nonlinear effects to be cumulative over the propagation distance.

Obviously, there are many ways to modify the ideal wedge geometry that give rise to dispersion of wedge acoustic waves. Such modifications may be undesired and due to damage or degradation, for example at the edges of turbine blades. Consequently, the dispersion of wedge acoustic waves can be made use of in non-destructive testing. On the other hand, weak dispersion can be introduced on purpose and the dispersion law tailored such that certain nonlinear effects like the formation of solitary pulses are favoured. Andersen, Datta and Gunshor discussed the possibility of modulational instability and the formation of envelope solitons in a regime of strong dispersion which impedes the growth of higher harmonics [15]. In both cases, a detailed understanding of the conditions leading to the dispersion of wedge acoustic waves is needed.

A considerable amount of experimental work was devoted to the investigation of the dispersion of wedge acoustic waves [16–31], including dispersion due to coating of a surface [28–30] or due to truncation of the wedge [18,20–23,25,26,31]. Theoretical studies were carried out mainly on the basis of thin plate theory [32] and the geometrical acoustics approximation [33–37] or by using the finite element method (FEM) [3,6,22,27,29,38,39]. Our investigations, which are based on perturbation theory and extend the early work of Lagasse, Cabus and Verplanken [40], supplement the results of ray theory, since the latter are strictly valid only in the regime of the geometrical lengths of the system much larger than the wavelength, while here, we are mostly dealing with the opposite limit.

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In this paper, the focus is on small dispersion, and among the many possible ways of modifying the wedge to generate dispersion, three different possibilities are analysed: (i) Coating of one or both of the two surfaces of the infinite wedge, (ii) truncating the wedge at its apex or replacing the tip of the wedge by a different material, (iii) slight modification of the material constants of the tip material in an extended spatial region of the wedge. Our goal is to establish the dispersion law, i.e. the dependence of frequency or phase velocity on inverse wavelength, in these three cases, and to produce quantitative results for the wavelength-dependent velocity shift of wedge acoustic waves with respect to the wedge wave velocity in ideal wedges. For this purpose, we use perturbation theory within the framework of an expansion of the displacement field in Laguerre functions [1,41–44]. In the third case, we also compare our results with FEM calculations. For investigations of small dispersion, the Laguerre function method has the advantage of not introducing any artificial length scale like the element size or the system size in FEM, that need to be controlled as they give rise to spurious dispersion of acoustic waves.

The paper is organised in the following way: in the next section, the theory of acoustic waves guided by perfect homogeneous elastic wedges, based on the Laguerre function method, is briefly reviewed, and notation is introduced that is made use of in the subsequent sections, where the perturbation analysis of the three dispersion cases is outlined, and results of analytic and numerical calculations are presented. A short discussion of the results concludes the paper.

2. Theory of wedge acoustic waves in anisotropic media

Calculations of the velocities of acoustic waves guided by wedges consisting of anisotropic elastic media were carried out with the help of the FEM (see e.g. [3]), the geometrical acoustics approximation [37] and on the basis of the Laguerre function method [43,44]. Here, we briefly describe the latter, starting with a coordinate system where the x-axis is along the apex of the wedge, the y-axis is parallel to one of the two surfaces, pointing from the apex in the direction of the medium, and the z-axis is normal to that surface, pointing in the inward direction of the medium (Fig. 1(a)). The fourth-rank tensor $(C_{\alpha\beta\mu\nu})$ of elastic moduli refers to this coordinate system. Here and in the following, Cartesian indices running from 1 to 3 are denoted by lower-case Greek letters, and summation over repeated Cartesian indices is implied. Cartesian indices that only run over 1 and 2 are denoted by upper-case Greek letters.

The equation of motion for the Cartesian components of the displacement field, u_α, $\alpha = 1, 2, 3$, reads

$$\rho \ddot{u}_\alpha = \frac{\partial}{\partial x_\beta} T_{\alpha\beta} \tag{2.1}$$

with density of the material ρ and stress tensor

$$T_{\alpha\beta} = C_{\alpha\beta\mu\nu} \frac{\partial u_\mu}{\partial x_\nu}. \tag{2.2}$$

At the two surfaces, the boundary conditions

$$N_\beta T_{\alpha\beta} = 0 \tag{2.3}$$

have to be satisfied, where N_α, $\alpha = 1, 2, 3$, are the three components of a vector normal to the corresponding surface. In addition, the displacement field has to decay to zero at large distances from the wedge tip.

Because of translational invariance along the x-direction (i.e. the material properties and the geometry of the wedge are independent of x), the displacement field may be set up in the form

$$u_\alpha(x, y, z, t) = \exp(i(kx - \omega t))\, w_\alpha(y, z|k) \tag{2.4}$$

involving the one-dimensional wavevector k and the frequency ω. The modal functions w_α, $\alpha = 1, 2, 3$, are approximated by an expansion in a set of functions $\{f_J(y, z), J = 1, 2, \ldots\}$ with expansion coefficients a_J^α. The latter depend on the elastic moduli and the density of the wedge material as well as on the wedge angle,

$$w_\alpha(y, z) = \sum_J a_J^\alpha f_J(y, z). \tag{2.5}$$

Inserting (2.4) with (2.5) in the equation of motion (2.1), projecting on function $f_I(y, z)$ (i.e. multiplying by $f_I(y, z)$ and integrating over the wedge's cross section) and making use of the boundary conditions (2.3), one is led to the generalized eigenvalue problem

$$\omega^2 \sum_K N_{IK} a_K^\alpha = \sum_J M_{IJ}^{\alpha\mu} a_J^\mu, \tag{2.6}$$

where (N_{JK}) is the mass matrix:

$$N_{JK} = \iint_A dydz\, f_J^*(y, z) f_K(y, z) \rho \tag{2.7}$$