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Characteristics of guided waves in anisotropic spherical curved plates

Yu Jiangong [a,*], Wu Bin [a], Huo Hongli [b], He Cunfu [a]

[a] College of Mechanical Engineering and Applied Electronic Technology, Beijing University of Technology, Ping Le Yuan 100#, Chaoyang District, Beijing 100022, PR China
[b] College of Water Conservancy and Hydroelectric, Hebei University of Engineering, Handan 056021, PR China

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Abstract

For ultrasonic non-destructive inspection of spherical curved plates, which is used in the fields of pressure vessels, spherical domes of power plants, and so on, characteristics of guided waves in spherical curved plates must be understood. Based on linear three-dimensional elasticity, an orthogonal polynomial series expansion approach is used for determining the guided wave dispersion curves and the displacements distributions in homogeneous anisotropic spherical curved plates. When the material is orthotropic or has fewer independent constants in the wave propagation direction, the coupled propagating waves can be decomposed of Lamb-like waves and SH wave. The independent SH wave is solved analytically by Bessel function. In order to test the accuracy and range of applicability of this polynomial approach, the two results of the SH wave is compared, the exact results by Bessel function and the results by the polynomial method. Guided wave dispersion curves for unidirectional composite spherical curved plates with different fiber intensity are calculated to show the effect of the fiber intensity on dispersion curves. Finally, the guided wave dispersion curves for orthotropic spherical curved plates at different radius of curvature are calculated and the inherited influences of the radius of curvature on the wave characteristics are discussed.

Keywords: Anisotropic spherical curved plate; Guided waves; Orthogonal polynomial; Radius of curvature; Dispersion curves

1. Introduction

As a new non-destructive test technology, guided waves have received a great deal of attention. Much work has been reported on the elastic wave propagation in cylindrical pressure vessels. However, similar attention has not been given to spherical plates, even though this study is important for inspecting spherical pressure vessels, domes and earth crusts. In 1966, Buldyrev and Lanin [1] investigated some aspects of wave propagation in solids with spherical boundaries. Brekhovskikh [2] also studied the surface wave propagation in solids

* Corresponding author. Tel.: +86 10 67391720 11.
E-mail address: yu@emails.bjut.edu.cn (Y. Jiangong).

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with curved boundary, in which cylindrical and spherical boundaries were considered as special cases. Using shell-theory, Shah et al. [3] analyzed the three-dimensional hollow spheres. Gaunaurd and Werby [4,5] derived dispersion curves for fluid loaded spherical shells. Kargl and Marston [6] also worked on the Lamb-like wave in isotropic spherical shells. Wang et al. [7] studied the stress wave propagation in orthotropic laminated spherical shells subjected to arbitrary radial dynamic load by means of finite Hankel transforms and Laplace transforms. Towfighi and Kundu [8] studied wave propagation in anisotropic spherical curved plates using the Fourier series expansion method.

In this paper, the Legendre orthogonal polynomial series expansion method, which was developed by Lefebvre et al. [9] for modeling free-ultrasonic waves in multilayered plates, and then for axial waves in anisotropic cylinders [10,11], is used to characterize guided waves in anisotropic spherical curved plates. The formulation is based on linear three-dimensional elasticity. Results are compared with those published earlier to confirm the accuracy and range of applicability of the developed Mathematica program. Guided wave dispersion curves for unidirectional composite spherical curved plates with different fiber intensity are calculated to show the effect of the fiber intensity on dispersion curves. The orthotropic spherical curved plates at different radius of curvature are analyzed and the inherited influences of the radius of curvature on the dispersion curves are discussed.

2. Mathematics and formulation of the problem

Based on linear three-dimensional elasticity, consider a hollow anisotropic sphere. In the spherical coordinate system (θ, ϕ, r), let a, b, h be the inner and outer radii, and the thickness.

Considering the boundary of the material, the position dependence of the elastic constants are given by

$$C(r) = C\pi(r), \quad \rho(r) = \rho\pi(r) \tag{1}$$

Where the $C(r)$ are position-dependent elastic constants of the medium that constitutes the hollow sphere and $\pi(r)$ is the rectangular window function defined by

$$\pi(r) = \begin{cases} 1, & a \leqslant r \leqslant b \\ 0, & \text{elsewhere} \end{cases}$$

Given Eq. (1), the material constants vanish outside the material. We thus describe the vacuum outside the material as a medium with zero acoustic impedance which ensures that the stresses outside the material vanish regardless of the displacement.

Then the generalized Hooke's law of an anisotropic material is given by

$$\begin{Bmatrix} T_{rr} \\ T_{\theta\theta} \\ T_{\varphi\varphi} \\ T_{\theta\varphi} \\ T_{r\varphi} \\ T_{r\theta} \end{Bmatrix} = \begin{bmatrix} C_{11}(r) & C_{12}(r) & C_{13}(r) & C_{14}(r) & C_{15}(r) & C_{16}(r) \\ & C_{22}(r) & C_{23}(r) & C_{24}(r) & C_{25}(r) & C_{26}(r) \\ & & C_{33}(r) & C_{34}(r) & C_{35}(r) & C_{36}(r) \\ & & & C_{44}(r) & C_{45}(r) & C_{46}(r) \\ & \text{symmetry} & & & C_{55}(r) & C_{56}(r) \\ & & & & & C_{66}(r) \end{bmatrix} \begin{Bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{\varphi\varphi} \\ 2\varepsilon_{\theta\varphi} \\ 2\varepsilon_{r\varphi} \\ 2\varepsilon_{r\theta} \end{Bmatrix} \tag{2}$$

Under the assumption of small deformations, the strain–displacement relationships in terms of the spherical coordinate system are given by

$$\begin{aligned} &\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, \quad \varepsilon_{\phi\phi} = \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r} + \frac{\cot\theta}{r}u_\theta, \\ &\varepsilon_{\theta\phi} = \frac{1}{2r}\left(\frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial \phi} + \frac{\partial u_\phi}{\partial \theta} - u_\phi \cot\theta\right), \quad \varepsilon_{r\phi} = \frac{1}{2r}\left(\frac{1}{\sin\theta}\frac{\partial u_r}{\partial \phi} - u_\phi\right) + \frac{1}{2}\frac{\partial u_\phi}{\partial r}, \\ &\varepsilon_{\phi\phi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) \end{aligned} \tag{3}$$

And the field equations governing wave propagation are