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Numerical implementation of fundamental solution for solving 2D transient poroelastodynamic problems

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Abstract

This paper presents the numerical implementation of boundary element formulation for solving two-dimensional poroelastodynamic problems in time domain. The derivation of the time-dependent integral equations is based on the Biot's theory and the reciprocal theorem. The analytical form of a 2D fundamental solution in time domain for porous media with incompressible components (solid particles and fluid) is derived and validated. After the analytical time integration of the fundamental solution kernels, a time-marching procedure is established. The comparison of different time interpolation functions shows that the mixed interpolation gives more stable response. In addition, the linear θ method is used in order to improve the numerical stability of the proposed approach. Finally, two examples are presented to investigate the stability and the accuracy of this approach for wave propagation analyses.

Keywords: Boundary element; Poroelasticity; Transient behaviour; Fundamental solution; Time-stepping; Stability

1. Introduction

The theory of continuous media for a wide range of fluid infiltrated materials, such as water saturated soils, or oil impregnated rocks, is a crude approximation for investigating the dynamic behaviour of theses materials. Indeed, the presence of a freely moving fluid in such materials may modify considerably their dynamic response. In these cases, the use of a theory, able to model the coupled mechanisms between the pore fluid and the solid skeleton, is necessary. This theory of poroelasticity, developed in an intuitive manner by Biot in a series of remarkable papers [1,2], is widely applied in civil engineering, geology, seismology, biomechanics, etc.

Because of the complexity of the governing equations, the resolution of poroelastic problems must resort to numerical methods, such the Finite Element Method (FEM) and the Boundary Element

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Method (BEM). The FEM, the most popular method, has been developed and applied with success in many problems of poroelasticity [3,4]. However, its major drawback is the difficulty in modelling infinite domains. The BEM, on the other hand, with its inherent ability to satisfy the far field conditions, is suitable for the class of problems involving infinite domains. The reduction of problem dimensionality is another advantage of the BEM.

Initially, the development of the BEM for poroelasticity was limited for quasi-static problems. Noteworthy publications include those of Cleary [5], Cheng & Ligget [6], Badmus et al. [7] for Laplace-domain formulations, and those of Dargush & Banerjee [8], Cheng and Detournay [9] for time-domain formulations. In the realm of dynamic poroelasticity, the BEM formulations have been firstly established by Manolis and Beskos [10], in the Laplace transform domain, and by Norris [11], in the frequency domain. However, the variables of this formulation, solid and average relative fluid displacements (u_i, w_i), are not practical for engineering purpose, because the measure of the relative fluid displacement w_i is difficult. In reality, the pore pressure p is commonly used instead of w_i. This quantity is directly measurable and consistent with soil and rock mechanics convention. In addition, it can be shown that, in transform domains, only the solid displacement u_i and the pore pressure p are independent [12]. Based on these unknowns, several integral formulations have been developed by Cheng et al. [13], Dominguez [14] for the frequency domain, and by Chen [15,16], Chen and Dargush [17], Gatmiri and Kamalian [18] for the Laplace domain. Note that, for these transform domain formulations, the time-domain response is derived by a subsequent inverse transformation from the transform domain response.

Another approach to model the transient behaviour of media by BEM is to deal directly with the time domain. Indeed, it is more natural to work in the real time domain and observe the phenomenon as it evolves. Generally, this approach consists of integrating the fundamental solution by a time-marching procedure. The first attempt to obtain the transient fundamental solution for dynamic poroelasticity was made by Burridge and Vargas [19]. Later, Wiebe and Antes [20] have found a time-domain solution by neglecting the viscous coupling, Kaynia and Banerjee [21] have derived an approximation of transient short-time solution. Nevertheless, the above solutions are sought for solid and fluid displacements (u_i, w_i). Based on four (three in 2D) unknowns (u_i, p), Chen [15,16] has proposed two approximations of the time-domain solution, namely the 'limiting case' and the 'general case', in which the second approach is in good agreement with the result obtained by the numerical inversion of Laplace domain solution. Afterward, Gatmiri and Kamalian [18] have suggested an approximate of long-time solution, neglecting the pore fluid acceleration. The above solutions are all approximate. By the knowledge of authors, until now, no exact analytical expression of the time-dependant fundamental solution was published.

In this paper, an integral formulation for poroelastodynamics in time domain is presented. In order to obtain the time-dependent fundamental solution, two assumptions are introduced. Firstly, the u–p formulation [3] is considered. Secondly, the solid particles and the fluid are assumed to be incompressible. According to these assumptions, an analytical transient fundamental solution are derived [22]. This formulation is appropriate for problems involving water saturated soils under earthquake solicitations. For the numerical implementation, three types of time interpolation functions are chosen so that time integrations can be performed analytically. The numerical instabilities of BEM solutions during the time-stepping procedure is observed. The mixed time interpolation is recommended because it gives a more stable solution. In addition, the linear θ method [23,24] is incorporated to improve the numerical stability.

2. Governing equations

The equations governing the transient response of poroelastic media can be expressed as follows [4]:

Equilibrium equation:

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i + \rho_f\ddot{w}_i \tag{1}$$

Constitutive relations:

$$\sigma_{ij} = \lambda u_{i,i} + \mu(u_{i,j} + u_{j,i}) - \alpha p\delta_{ij} \tag{2}$$

$$p = M(\zeta - \alpha u_{i,i}) \tag{3}$$