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Phononic properties of hexagonal chiral lattices

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ABSTRACT

The manuscript reports the outcome of investigations on the phononic properties of a chiral cellular structure. The considered geometry features in-plane hexagonal symmetry, whereby circular nodes are connected through six ligaments tangent to the nodes themselves. In-plane wave propagation is analyzed through the application of Bloch theorem, which is employed to predict two-dimensional dispersion relations as well as illustrate dispersion properties unique to the considered chiral configuration. Attention is devoted to determining the influence of unit cell geometry on dispersion, band gap occurrence and wave directionality. Results suggest cellular lattices as potential building blocks for the design of meta-materials of interest for acoustic wave-guiding applications.

1. Introduction

Cellular solids, while long known and commonly exploited for disparate uses in traditional engineering applications, currently offer new opportunities to design structured materials capable of delivering unique elastic, electromagnetic, and thermal properties. The behavior of cellular materials, is strongly dependent upon their topology or spatial arrangement, providing ample opportunities in the design of novel structured configurations. Examples of unusual behavior, within the realm of solid mechanics, include auxetic or negative Poisson's ratio characteristics of re-entrant, hexagonal honeycombs, structured materials which expand perpendicularly to applied uniaxial elongation, in contrast to "classical" solids [1,2]. Periodic cellular materials are defined by the spatial repetition of an irreducible geometric domain or unit cell, and their topology, moreover, strongly affects the propagation characteristics of elastic waves. The periodicity or group of a given lattice, in fact, determines frequency bands within which the propagation of elastic waves is permitted (pass bands) or forbidden (band gaps or stop bands). Anisotropy in the elasto-dynamic behavior of periodic structural assemblies, furthermore, can be exploited to steer or guide waves in specific directions (beaming) [3,4].

Significant control afforded by structural lattices on phenomena such as band gaps and wave-beaming, owing to the large number of possible configurations and advances in manufacturing capabilities, suggest their employment as alternatives to elastic composites with periodic mass and stiffness modulations, typically known as phononic crystals. The selection of the periodic material distribution in phononic crystals, in particular, is based on the need to generate band gaps at specified frequencies, and/or to guide elastic waves along a desired path [5]. A thorough classification of unit cell classes of periodic cellular structures and associated elasto-dynamic behavior, already undertaken by [3,4] for such topologies as the hexagonal, triangular, and Kagomé lattices to name a few, may provide the possibility of designing structured materials with easily selectable wave propagation behavior.

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The unique characteristics of phononic materials are currently exploited in a number of applications which include sensing devices based on resonators, acoustic logic ports, wave guides and filters based on surface acoustic waves (SAW). Synthesis of phononic materials with desired band gap and wave-guiding characteristics has achieved promising maturity, mostly through the application of topology and material optimization procedures [6–8]. Although very effective, such techniques may require intensive computations and may lead to complex geometries difficult to manufacture and whose performance may lack in robustness. The application of periodic structural networks as phononic materials may potentially lead to a simplified design process, where a limited number of continuously varying parameters defines the geometry of a predefined cellular topology [9]. The successful application of this approach clearly requires identifying lattice configurations that provide sufficient design flexibility, as well as with strongly dependent elasto-dynamic properties upon a limited number of geometric parameters defining the lattice configuration.

This observation motivates the investigation of the wave propagation characteristics of the chiral structural lattice considered in this paper. The design flexibility and the unique properties of this cellular configuration, originally proposed by [10,11], and documented in a number of recent papers by the authors [12,13], are here evaluated in terms of wave propagation characteristics with the intent of broadening the already promising features offered by more traditional configurations such as triangular and hexagonal lattices proposed by [3,4]. The influence of the unit cell configuration on band gaps and wave-guiding properties is investigated through a numerical model constructed considering chiral cellular topologies as assemblies of beams connected to form a frame structure. Specifically, a compact description of the dynamic behavior of the chiral assembly is provided by an elasto-dynamic discretization based on Timoshenko beam elements [14]. Finally Bloch Theorem is employed to obtain dispersion surfaces, band diagrams, and to investigate the dependency of group speeds and phase velocities on frequency and direction of wave propagation.

The present paper is organized in five sections including the introduction above. Section 2 describes the considered lattice configuration and its geometric properties, while Section 3 describes the approach used for the analysis of free wave propagation in the lattice. Section 4 presents results in terms of dispersion surfaces, band diagrams and wave velocities and discusses their sensitivity with respect to a set of characteristic parameters for the lattice. Finally, Section 5 summarizes the objectives of the work and its main results.

2. Geometry of a hexagonal chiral lattice

2.1. Geometric parameters and properties

The structural layout of a hexagonal chiral lattice, shown in Fig. 1, consists of circular elements of radius r, acting as nodes, connected by ribs or ligaments, of length L tangent to the nodes themselves. The distance between node centers is denoted as R, while the angle between the imaginary line connecting the node centers and the ribs is defined as β. The angle between adjacent ligaments is denoted as 2θ. Finally, the wall thickness of nodes and ribs is denoted as t_c and t_b, respectively. As described in [10], the following geometric relationships hold:

$$\sin\beta = \frac{2r}{R}, \quad \tan\beta = \frac{2r}{L}, \quad \sin\theta = \frac{R/2}{R}, \quad \cos\beta = \frac{L}{R}. \tag{1}$$

The ratio L/R yields significantly different configurations, as depicted in Fig. 2, and thus is here denoted as the *topology parameter*. The unit cell of the honeycomb depicted in Fig. 2[1] is highlighted in dashed red lines, and it constitutes the smallest structural domain that encompasses the complete set of geometric entities necessary to analyze the lattice's mechanical behavior. Notably, the possible configurations obtained for variations of the topology parameter span those composed of packed circles ($L/R \to 0$) to triangular assemblies ($L/R \to 1$ or $L/r \to \infty$).

An additional geometric property of the hexagonal chiral lattice, as its name indicates, is in-plane hexagonal symmetry. As demonstrated by the third of Eq. (1), the angle θ is always 30°, indicating that the hexagonal chiral topology is invariant to in-plane rotations by the angle 2θ, regardless of the topology parameter. The influence of hexagonal symmetry on the elastostatic behavior results in a mechanical condition of in-plane isotropy [15]. The Young's modulus of a hexagonal material, as a result, is independent of direction in the plane normal to the hexagonal symmetry axis [16]. The chiral lattice, furthermore, features a negative in-plane Poisson's ratio of the order of ≈ -1 [10], and design flexibility resulting from the considerable sensitivity of mechanical behavior to the unit cell parameters L, R, θ, t_b, and t_c. For example, the static compliance of the assembly can be significantly modified by varying the topology parameter [12,13].

2.2. Unit cell configuration and lattice vectors

The structural lattice under investigation is obtained from the assembly of unit cells of the kind shown in Fig. 3. As in any periodic assembly, the location of a generic point can be described in terms of location within a reference unit cell and a set of lattice vectors which define the periodicity of the system. Introducing a reference frame in the plane of the lattice $\mathscr{F}_{\mathscr{I}}$ defined by an orthogonal unit vector basis $\mathscr{I} = (\boldsymbol{i}_1, \boldsymbol{i}_2)$, the location of a point P in cell n_1, n_2 can be expressed as:

$$\boldsymbol{\rho}_P(n_1, n_2) = \boldsymbol{r}_P + n_1\boldsymbol{e}_1 + n_2\boldsymbol{e}_2, \tag{2}$$

[1] For interpretation of color in Fig. 2, the reader is referred to the web version of this article.