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# Effective pulse dynamics in optical fibers with polarization mode dispersion

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**Abstract**

This paper investigates pulse propagation in randomly birefringent optical fibers. We consider two polarization-mode dispersion models. Using a separation of scales technique we derive an effective stochastic partial differential equation for the envelope of the field. This equation is driven by three independent Brownian motions, and it depends on the polarization-mode dispersion model through a single effective parameter. This shows that pulse dynamics in randomly birefringent fibers does not depend on the microscopic model. Numerical simulations are in excellent agreement with the theoretical predictions.


*Keywords:* Optical fiber; Polarization mode dispersion; Waves in random media

## 1. Introduction

Optical fibers have been extensively studied because they play an important role in modern communication systems [1]. In particular, polarization mode dispersion (PMD) has attracted the attention of engineers, physicists and, more recently, applied mathematicians. PMD is one of the main limiting effects of high bit rate transmission in optical fiber links. It has its origin in the birefringence [2,3], i.e. the fact that the electric field is a vector field and the index of refraction of the medium depends on the polarization state (i.e. the unit vector pointing in the direction of the electric vector field). For a fixed position in the fiber, there are two orthogonal polarization states which correspond to the maximum and the minimum propagation velocities. These two polarization states are parameterized by an angle with respect to a fixed pair of axes that is called the birefringence angle. The difference between the maximum and the minimum of the index of refraction is the so-called birefringence strength. If the birefringence angle and strength were constant along the fiber, then a pulse polarized along one of the eigenstates would travel at constant velocity. However the birefringence angle is randomly varying which involves coupling between the two polarized modes. The modes travel with different

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velocities, which involves pulse spreading. Random birefringence results from variations of the fiber parameters such as the core radius or geometry. There exist various physical reasons for the fluctuations of the fiber parameters. They may be generated during the drawing process as a result of a drift of the operating parameters. They may also be induced by mechanical distortions on fibers in practical use, such as point-like pressures or twists [4].

In our paper we consider the linear propagation of a pulse in a randomly birefringent optical fiber. The electric field is solution of two coupled Schrödinger equations with random coefficients. Randomness is inherent in PMD. The length scales present in the problem are the fiber length, the beat length (proportional to the inverse of the birefringence strength), and the PMD correlation length. We consider realistic configurations where the fiber length (typically $10^3$ km) is larger than the PMD correlation length (typically a few tens of meters) which is itself larger than the beat length (a meter or less) [5]. We study the asymptotic behavior of the field envelope in the framework based on the separation of these scales, using a technique first introduced by Papanicolaou and coworkers [6]. We derive an asymptotic stochastic partial differential equation driven by three independent Brownian motions and parameterized by a single effective PMD parameter. This result can be extended to more general configurations, where other length scales come into the play: the nonlinear length and the length of the dispersion map in the presence of a dispersion-management scheme. The result obtained in this paper should remain true as long as the PMD correlation length is smaller than these new length scales, which holds true in practical configurations. We shall give numerical evidence of this conjecture.

The paper is organized as follows. Section 2 is devoted to the presentation of the first model introduced by Wai and Menyuk [2,3] for linear randomly birefringent optical fibers. In Section 3 we introduce the second model introduced by Wai and Menyuk [2,3]. Section 4 is devoted to the study of the asymptotic dynamics of the electric field in the two PMD models. In Section 5 we present numerical results which are compared with the theoretical asymptotic results.

## 2. Random birefringence with constant birefringence strength

The evolution of the two polarization modes of the electric field in a randomly birefringent fiber is governed by the coupled Schrödinger equation [2]:

$$\mathrm{i}\frac{\partial E}{\partial z} + bK(z)E + \mathrm{i}b'K(z)\frac{\partial E}{\partial t} + \frac{d_0}{2}\frac{\partial^2 E}{\partial t^2} = 0, \tag{1}$$

where $E(z,t)$ is the column vector $(E_1(z,t),E_2(z,t))^{\mathrm{T}}$ representing the field vector envelope depending on the position $z \in [0,+\infty)$ and the local time $t \in \mathbb{R}$. $b$ (resp. $b'$) is the birefringence strength (resp. the frequency-derivative of the birefringence strength). The usual beat length is related to $b$ through the identity $L_B = \pi/b$. We denote by $\theta(z)$ the random birefringence angle which depends on the position $z$. The matrix $K$ is given by

$$K(z) = \cos\theta(z)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sin\theta(z)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

$d_0$ is the group velocity dispersion (GVD) parameter. We perform the change of variables $\widetilde{E} = M^{-1}(z)N^{-1}(z)E$ where

$$N(z) = \begin{pmatrix} \cos(\theta(z)/2) & -\sin(\theta(z)/2) \\ \sin(\theta(z)/2) & \cos(\theta(z)/2) \end{pmatrix} \quad \text{and } M(z) = \begin{pmatrix} \widetilde{v}_1(z) & \widetilde{v}_2(z) \\ -\overline{\widetilde{v}_2}(z) & \overline{\widetilde{v}_1}(z) \end{pmatrix}$$

is the unitary matrix solution of

$$\mathrm{i}\frac{\mathrm{d}M}{\mathrm{d}z} + \left[\frac{\mathrm{i}}{2}\frac{\mathrm{d}\theta}{\mathrm{d}z}(z)\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + b\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]M = 0.$$

Here $\bar{x}$ stands for the conjugate of a complex number $x$. The equation describing the evolution of the new electric field $\widetilde{E}$ is:

$$\mathrm{i}\frac{\partial \widetilde{E}}{\partial z} + \frac{d_0}{2}\frac{\partial^2 \widetilde{E}}{\partial t^2} + \mathrm{i}b'\widetilde{\Sigma}(z)\frac{\partial \widetilde{E}}{\partial t} = 0, \tag{2}$$