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Iterative methods for scattering problems in isotropic or anisotropic elastic waveguides

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HIGHLIGHTS

- We derive new families of modal transparent boundary conditions for elastic waveguides, which have not been considered in the literature.
- The benefit of using an overlap between the finite element domain and the modal domain is emphasized.
- We construct an original and particularly efficient transparent boundary condition.
- This transmission condition enhances the effect of the overlap and allows to handle arbitrary anisotropic materials.

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ABSTRACT

We consider the time-harmonic problem of the diffraction of an incident propagative mode by a localized defect, in an infinite elastic waveguide. We propose several iterative algorithms to compute an approximate solution of the problem, using a classical finite element discretization in a small area around the perturbation, and a modal expansion in the unbounded straight parts of the guide. Each algorithm can be related to a so-called domain decomposition method, with an overlap between the domains. Specific transmission conditions are used, so that at each step of the algorithm only the sparse finite element matrix has to be inverted, the modal expansion being obtained by a simple projection, using a bi-orthogonality relation. The benefit of using an overlap between the finite element domain and the modal domain is emphasized. An original choice of transmission conditions is proposed which enhances the effect of the overlap and allows us to handle arbitrary anisotropic materials. As a by-product, we derive transparent boundary conditions for an arbitrary anisotropic waveguide. The transparency of these new boundary conditions is checked for two- and three-dimensional anisotropic waveguides. Finally, in the isotropic case, numerical validation for two- and three-dimensional waveguides illustrates the efficiency of the new approach, compared to other existing methods, in terms of number of iterations and CPU time.

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1. Introduction

The development of non destructive testing techniques using ultrasonic guided waves (see [1,2] and the references herein) motivates the improvement of existing numerical methods of simulation. In particular, efficient methods are required to compute the scattering of guided waves by arbitrary defects in elastic waveguides. Classically, the waveguide is supposed to be infinite and perfectly uniform, except in a bounded area containing the defect. A natural objective is to reduce the finite element computations to a region as close as possible to this perturbed area. The difficulty is then to handle the artificial boundaries of the finite element domain in order to avoid spurious reflections. This is an old problem [3] which has been satisfactorily solved in case of scalar equations, but still raises open questions for vectorial equations arising in electromagnetic or elastic waveguides. For scalar problems, two classes of methods can be used. Let us explain for each of them what are the specific issues when considering elastic waveguides.

A first class of methods consists in putting on each side of the perturbed area a perfectly matched absorbing layer (PML), so that the computed diffracted field almost vanishes at the end of the layer. This technique is easy to integrate in any Finite Element code (no specific implementation is needed) and leads us to solve a classical sparse linear system. Unfortunately, it is well-known that PMLs do not work in elastic waveguides [4] because of the existence in some range of frequencies of backward modes, whose group and phase velocities are of opposite signs. In such configuration, the PMLs do not select the correct outgoing solution. A remedy has been proposed and analyzed in [5] where the physical solution is reconstructed a posteriori by combining several wrong fields computed with PMLs. An alternative consists in using adiabatic viscoelastic absorbing layers [6] which are not perfectly matched and need to be sufficiently large to avoid spurious reflections. The main drawback of this approach is then its computational cost. Also let us point out that absorbing layer techniques (perfectly matched or not) require a fine adjustment of some parameters, which may limit their systematic use in an industrial context. Let us finally mention a more recent method for elastic waveguides based on Hardy space infinite elements [7] whose development is still in progress.

A second possibility consists in using the modal decomposition of the field outside the perturbed area to derive transparent boundary conditions on the artificial boundaries of the finite element domain. The advantage is that such conditions are exact (and with an exponentially small error at the discrete level if enough modes are kept in the modal expansion). Different ways to implement such conditions have been proposed in the literature. In [8,9], a formulation involving both finite elements and modal unknowns is derived, while only one type of unknowns is generally kept, modal unknowns in [10,11] and finite elements unknowns in [12]. For the latter, the difficulty is due to a lack of orthogonality of the displacement fields associated with elastic modes. As a consequence, it is not possible to obtain a diagonal expression of the natural Dirichlet to Neumann operator (relating the normal stress to the displacement) in elastic waveguides. In the isotropic case, an alternative has been proposed in [12] (see [13] for more details and [14,15] for applications), where the authors derived transparent boundary conditions relating hybrid displacement/stress vectors. This work is based on a bi-orthogonality relation, mixing displacement and stress components (which has been derived first by Fraser in 2D [16] and then extended to the 3D case [17]). Let us point out that a scalar Lagrange multiplier has to be introduced on the artificial boundaries, because these transparent conditions are not naturally compatible with the variational formulation in the perturbed area. The method is very accurate but requires a specific implementation and leads to a partially dense linear system. Such system can be difficult to invert, in particular for elastic 3D configurations. Moreover, this approach cannot be used in the general anisotropic case.

In our work, we intend to gather advantages of both classes of methods. In other words, we would like to design a method using transparent boundary conditions based on modal expansions and such that only simple and sparse systems have to be inverted. A natural idea is to use iterative algorithms instead of direct ones to solve a system involving transparent boundary conditions. This framework has been already applied to several problems set in unbounded domains [18–20]. One of the main features is that the system to invert (or equivalently the preconditioner) is chosen as a sparse part of the complete system. As a consequence, the dense part of the matrix coming from the non-local transparent condition is involved only in the matrix–vector product step. It is also instructive to relate these iterative algorithms to domain decomposition methods. The specificity here is that only two subdomains are introduced, a bounded one and an unbounded one where the equation can be explicitly solved, using an analytical representation (a modal expansion or an integral representation for instance). Different algorithms are derived by modifying the transmission conditions between the two subdomains, which can overlap or not. It is now well known that algorithms of this type do not converge in general for time harmonic wave equations [20] but they can be used to design preconditioned Krylov methods like GMRES [21]. The main criterion to discriminate between different algorithms is the rate of convergence of the associated GMRES algorithms.

In the present paper, we want to adapt these ideas to the case of elastic waveguides, which has not been considered in the literature. Combining such point of view with the ideas of [12], we derive new families of modal transparent boundary conditions. The benefit of adding an overlap is emphasized. Besides classical effects (for instance improvement of the convergence rate of associated iterative algorithms), it allows the construction of a particularly efficient transparent boundary condition described in Section 4. Last but not least, this condition can be used for anisotropic waveguides. This is the main contribution of the paper.

The paper is organized as follows. In Section 2, the main notions concerning elastic modes in the isotropic case are summarized. Then, we introduce a well-posed boundary value problem in a semi-infinite straight waveguide, with a modal expansion of its solution. Finally, the scattering problem we are interested in is defined. In Section 3, domain decomposition