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Wave dispersion in randomly layered materials

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ABSTRACT

Wave scattering in materials composed of two kinds of alternating layers with different elastic properties and randomly distributed thicknesses has been modeled. The general form of the dispersion equation is derived for the unbounded layered medium. It defines two basic macroscopic characteristics of the scattered wave: phase velocity and attenuation, which are explicit functions of wave frequency and microscopic parameters of the system: acoustic properties of the layers and stochastic characteristics of their thickness distributions. The analytical expressions are derived for three special cases: for long waves; for a periodic medium composed of layers with constant thicknesses and for random medium with uniform distribution of layer thicknesses. Special attention is paid to the analysis of the frequency dependence of the wave parameters. It was shown that the predictions of the model for long waves and for periodic medium are compatible with the results obtained in the literature.

Moreover, comparison of theoretical results for frequency dependent wave parameters with numerical simulations of pulse transmission through the slab of the randomly layered medium shows good qualitative and quantitative agreement in wide frequency range.

1. Introduction

Wave scattering is a complex phenomenon that occurs when waves propagate in heterogeneous materials such as rocks, concrete and biological tissues, and is the subject of studies in many areas, e.g.: geophysics, non-invasive and non-destructive testing, medical imaging and diagnosis. Because of the dynamic and non-local character of this phenomenon and its strong frequency dependence, modeling of the scattering is difficult and requires the use of advanced mathematical methods. This causes a great interest in studies of wave scattering in systems where complex internal structure of heterogeneous materials is approximated by simplified geometries, e.g., layered structures, spherical cavities and inclusions, rods or cylindrical tubes [1–12]. A review of applications of various simplified models for prediction of wave scattering by trabecular bone is presented in the paper by Wear [8]. These include the Faran cylinder model, thin cylinder model, weak scattering model and other approaches based on the multiple scattering. Hughes et al. [10,13] suggested that even a simple periodic two-layered medium might provide an alternative model for description of wave propagation phenomena in trabecular bone. In the paper by Hudson and Heritage [9], complex phenomena of seismic wave scattering caused by small-scale heterogeneities in the Earth is described by the simplified model based on the single-scattering perturbation theory (the Born approximation).

The problem of elastic wave scattering in various layered materials has been considered by many authors [3,4,6,7,10,11,14–17]. Gilbert and Backus [3,15] proposed a general synthetic method for description of wave propagation in the stratified media, called the matrix propagator method, which is used by many authors in seismic applications. Despite the generality

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of this method, the explicit expressions for the parameters of wave propagation were derived only for the simple case of the periodic layered medium, and have been used to investigate several limiting cases. Fouque et al., in their monograph [4], present extensive studies of wave propagation and time reversal in randomly layered media using the formalism of the propagation matrix and homogenization theory. A quantitative mathematical analysis, performed in the book, is mainly aimed at understanding the surprising effects of enhanced spatial focusing and time compression in random media observed during time-reversal experiments.

The main objectives of the present paper is to formulate a general equation for wave dispersion in layered media composed of two kinds of alternating layers having randomly distributed thicknesses and different acoustic properties, to derive analytical expressions describing wave characteristics for special cases of the dispersion equation, and to analyze their dependence on wave frequency and parameters characterizing acoustic and random properties of layers.

Theoretical considerations presented in the paper are based on the structural approach in which the path structure of wave propagation in the system is analyzed and can be easily represented in the form of graphic schemes. This allows direct formulation of equations for the resultant amplitudes of waves propagating in the system, without the need for analysis of their multiple reflections and transmissions through contact surfaces of layers. This approach also takes into account the fact that the acoustic properties of the whole system are determined by the wavenumbers and acoustic impedances for the layers and by their geometrical characteristics. The similar approach was used in papers [18–20]. Lytvynenko et al. [18,19] applied this method to the analysis of electromagnetic wave diffraction in multi-layered medium of periodic structure whereas Cieszko et al. [20] used this approach to derive differential equations for monochromatic wave propagation in continuously inhomogeneous materials. In the present paper the structural approach is extended for description of wave propagation in the randomly layered medium.

Applying the structural approach, equations for reflection coefficients and wave transfer coefficients through the surface layer of randomly layered half-spaces were formulated. Analysis of the system of equations for the wave transfer coefficients enabled derivation of the general form of the dispersion equation for unbounded layered medium. It defines two basic macroscopic characteristics of the scattered wave: phase velocity and attenuation, which are explicit functions of wave frequency and microscopic parameters of the system: acoustic properties of the layers and stochastic characteristics of their thickness distributions. The dispersion equation was used to derive the analytical expressions describing wave characteristics for three special cases: for long waves; for a periodic medium composed of layers with constant thicknesses; and for random medium with uniform distributions of layer thicknesses. Special attention was paid to the analysis of frequency dependence of wave parameters for materials characterized by high and low contrast of the acoustic properties of the layers. It was shown that the predictions of the proposed model for long waves and for periodic medium are compatible with the results obtained by other authors [10,11]. It was proved that in the long wavelength range phase velocity and attenuation of waves propagating in the layered medium with random distribution of layer thicknesses are well approximated by these parameters for the periodic layered medium in which thicknesses of layers are replaced by the mean thicknesses of layers in the randomly layered medium. The approximation is the better, the lower are deviations of the layer thicknesses in the random medium. It was shown also that in layered media with low contrast of acoustic properties of the layers the wave attenuation coefficient is proportional to the second power of frequency, and the constant coefficient in this relation is a square function of deviations of layer thicknesses.

The obtained theoretical results for frequency characteristics of wave propagation in the randomly layered medium with uniform distributions of layer thicknesses are compared with such characteristics determined from computer simulation data of impulse wave propagation in macro layer of the randomly layered medium. Considerations are based on the ergodic hypothesis, according to which the estimates of stochastic process obtained by sample averaging converge to their ensemble averages when the size of the sample increases. It was shown that attenuation coefficient and phase velocity of wave determined in simulations are in good agreement with the theoretical characteristics in wide frequency range.

2. Modeling of wave interaction with randomly layered half-spaces

2.1. Formulation of the problem

The problem of plane harmonic wave interaction with layered half-space composed of two kinds of alternating layers is considered. The layers have randomly distributed thicknesses and different acoustic properties (see Fig. 1). It is assumed that the materials of the layers in the layered half-space ($x > 0$) and of the adjoining homogeneous half-space ($x < 0$) are elastic and perfectly joined, and that the wave impinges perpendicularly upon their contact surface.

The randomly layered medium (composed of two materials) may form two kinds of half-spaces with different acoustic properties. We call them half-spaces of type I and type II, depending on the type of material which forms the first layer (denoted by the subscript 1 or 2). We also assume that the distributions of the layer thickness u of both materials are defined by the probability density functions $f_1(u)$ and $f_2(u)$, respectively, and the homogeneous half-space has the same acoustic properties as the second layer of the adjoining layered half-space. This allows derivation of the system of equations for parameters describing wave interaction with the randomly layered half-space.

The basic acoustic characteristic of the analyzed system is the parameter α defined as ratio of the acoustic impedances of both layers (z_1, z_2):

$$\alpha = z_1/z_2 \tag{2.1}$$