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Application of plane wave expansion and stiffness matrix methods to study transmission properties and guided mode of phononic plates

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HIGHLIGHTS

- Generalization of the stiffness matrix method for bidirectional phononic plates.
- Agreement was found between the band structure and the transmission spectrum.
- Guided modes appear in stop band when a homogeneous layer replaces phononic layer.

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ABSTRACT

Numerical procedure based on plane wave expansion and stiffness matrix method is developed to calculate the transmission factor of a micro two-dimensional phononic plate. Calculations of the dispersion curve have been achieved by introducing particular functions which transform motion equations into an eigenvalue problem. The state vector has been generalized to a phononic material, it leads to a comparatively convenient matrix formulation. The influence of the layer number on the transmission factor is studied. In addition, our interest is focused on the observed gap and how it behaves when phononic structure undergoes a slight change. The result shows that if the central phononic layer is replaced by one or two homogeneous layers, guided modes originate inside the frequency band gaps.

1. Introduction

The propagation of elastic waves in periodic heterogeneous materials which is called phononic crystals (PCs) has received much attention in the last fifteen years [1–4]. One of the properties of these composite materials is the possibility of having phononic band gaps, in which vibrations at certain frequencies do not propagate.

These materials are of great interest for many applications, such as transducers, elastic acoustic filters, noise control, and vibration shields. Most of previous work concentrates on PCs made of elastic isotropic materials; however, band gaps can be enlarged by using non-isotropic materials, such as piezoelectric materials [5]. At the micro-scale, phononic crystals are useful for acoustic isolation of vibrating structures [6].

To study the elastic wave behavior in this kind of system, several numerical analytical methods such as the plane wave expansion method (PWE), [1,2,7–9] the multi-scattering theory (MST) [10], and the finite-different time-domain method

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Table 1
Mechanical characteristics and transverse velocity of the components of phononic materials.

Material	Si	W
ρ (kg/m^3)	2329	19260
V_T (m/s)	5842.5	2887.7
C_{11} (10^{11} Pa)	1.656	5.224
C_{12} (10^{11} Pa)	0.639	2.044
C_{44} (10^{11} Pa)	0.795	1.606

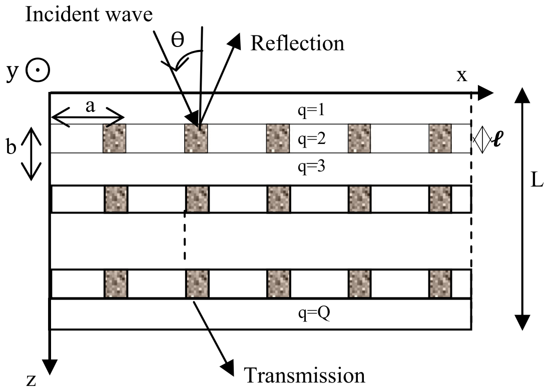

Fig. 1. Phononic plate geometry: a, b are the lattice constants and ℓ the width of the implantation, θ is the incident angle.

(FDTD) [11] have been developed and extensively used. Among them, the PWE method, by which the wave equations are solved in the Fourier space, is mostly used to calculate the band structure.

The purpose of the present study is to elucidate a calculating method of transmission and reflection spectrum through 2D phononic plate. Zhilin Hou et al. used the eigenmode matching theory (EMMT) [12]. In our case, numerical procedure based on plane wave expansion (P.W.E) and stiffness matrix method (SMM) [13–16] is developed to calculate the transmission and reflection factors of a micro two-dimensional phononic plate. To our knowledge, the SMM which is developed by Wang, Rajapakse and Rokhlin and used in the study of multilayer is never used in the case of phononic materials.

We cut the system into homogeneous and phononic layers, applied the Fourier transformation and the plane wave expansion in each layer, then we applied stiffness matrix method to connect the nearest layers by application of the boundary conditions. This method is convenient because it allows avoiding problems of divergence and permits to calculate transmission and reflection spectra. Computation results are checked and the validation is achieved on Z. Hou's work [12].

For illustration, a two-dimensional square lattice consisting of Tungsten (W) rectangular bars embedded in a Silicon (Si) matrix is investigated. The choice of W and Si is related to the fact that these materials present a great contrast requisite to their mechanical characteristics (Table 1). In addition, W/Si is a good candidate to micro-phononic crystal fabrication [6].

Our interest is focused on the observed gap and how it behaves when phononic structure undergoes a slight change. The creation of defect in the plate by replacing the central layer phononic by one or two homogeneous layers leads to the appearance of one or two very fine modes inside the band gap known as guided modes.

2. Phononic materials

We consider a two-dimensional phononic crystal, as shown in Fig. 1, with lattice constants (a and b) and filling fraction $f = \frac{\ell^2}{ab}$, where ℓ represents the width of the square implantation. The material is infinite according to the x and y directions but finite along z direction. To study the transmission spectrum through a phononic plate, whose thickness is L, we can denote the considered system by layer labels $q = 1, 2, 3, \ldots, Q$ in z-direction, where the uniform and 1D phononic layers are interlaced. The elastic plane wave is imputed from the most top layer 1 and outgoing from the most down layer Q. θ is the incident angle. Materials of incidence and transmission environment are considered homogeneous.

3. Basic equations

In order to obtain elastic wave propagating in phononic plate, we first applied PWE method to express the waves in each layer by elementary functions, and then we determined the stiffness matrix to apply the boundary conditions. The elastic wave propagating in elastic media can be described by those two equations:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} \quad (1)$$