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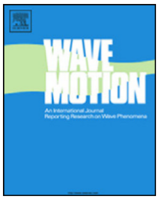

Polynomial approach for modeling a piezoelectric disc resonator partially covered with electrodes

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HIGHLIGHTS

- The method has been extended to model high contrast electromechanical devices.
- The method is applied to the modelling of a partially electroded disk resonator.
- Our model calculates normal frequencies, electric input admittance and of all modes.
- Our model calculates also electromechanical coupling coefficients of all modes.

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ABSTRACT

The frequency spectrum of a partially metallized piezoelectric disc resonator was studied using Legendre polynomials. The formulation, based on three-dimensional equations of linear elasticity, takes into account the high contrast between the electroded and non-electroded regions. The mechanical displacement components and the electrical potential were expanded in a double series of orthonormal functions and were introduced into the equations governing wave propagation in piezoelectric media. The boundary and continuity conditions were automatically incorporated into the equations of motion by assuming position-dependent physical material constants or delta-functions. The incorporation of electrical sources is illustrated. Structure symmetry was used to reduce the number of unknowns. The vibration characteristics of the piezoelectric discs were analyzed using a three-dimensional modelling approach with modal and harmonic analyses. The numerical results are presented as resonance and anti-resonance frequencies, electric input admittance, electromechanical coupling coefficient and field profiles of fully and partially metallized PIC151 and PZT5A resonator discs. In order to validate our model, the results obtained were compared with those published previously and those obtained using an analytical approach.

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1. Introduction

Micro-electro-mechanical systems (MEMS) resonators are rapidly gaining in importance as they are being used in an increasing number of different fields of application such as transportation, communication, automated manufacturing, environmental monitoring, healthcare, and defense systems, as well as in a wide range of consumer products [1–7]. Among the different types of MEMS, resonantly driven micro-devices are an important branch requiring the analysis of the resonance and anti-resonance frequencies and the modal shapes of the structure [8].

The vibration characteristics of piezoelectric structures are determined entirely from the three-dimensional equations of linear elasticity, the Maxwell equations, and the piezoelectric constitutive equations [9,10]. Numerous studies deal with modelling MEMS piezoelectric resonators. Guo et al. [11] presented and calculated the resonance frequencies of PZT-5A piezoelectric discs with diameter-to-thickness ratios of 20 and 10 using their vibration characteristics. Ivina [12] analysed the symmetric thickness vibrations of piezoelectric discs with partial axisymmetric electrodes using the finite element method. Schmidt [13] employed linear piezoelectric equations to investigate the extensional vibrations of a thin, partly electroded piezoelectric plate. Rogacheva [14] used the finite element method to analyse and calculate the resonance and anti-resonance frequencies and electromechanical coupling coefficients of piezoceramic discs and cylindrical shells. Chi-Hung Huang [15] analysed a thin piezoceramic disc partially covered with electrodes using linear theoretical and experimental vibration.

In the literature, several methods have been used to model acoustic propagation in piezoelectric structures. Among these, the Legendre polynomial method provides excellent precision for waveguides of various geometries such as planar and cylindrical multilayered and functionally graded structures [16–19]. This method uses constitutive and propagation equations to describe the structure, and is easy to program. The boundary conditions are automatically incorporated in the equations of motion using position-dependent physical constants, stiffness, permittivity, piezoelectric tensors and mass density [20–22]. Moreover, the acoustic field distributions are easily obtained [23–25]. However, (i) it has only been applied to modelling Bulk Acoustic Wave (BAW) resonators in plates [26–28], never to analyse partially electroded cylindrical resonators; (ii) its convergence depends on the relative properties of the materials.

In this paper, as announced in the prospects of a previous paper [25], we present a polynomial approach for studying the frequency spectrum of a partially electroded piezoelectric MEMS resonator disc. The formulation statement is based on three-dimensional linear elasticity using an analytic form for the field variables. According to the geometry of the structure, the boundary, symmetry, and continuity conditions are automatically incorporated in the physical equations that govern the structure. The incorporation of the electrical source in the field equations is illustrated. The numerical results for harmonic and modal analyses are presented for full and partial metallization. To take into account the high contrast between the electroded and non-electroded regions, the structure studied was divided into two parts: the electroded one and the non-electroded one. Resonance and anti-resonance frequencies, electric input admittance (impedance), electromechanical coupling coefficient and field profiles, which are easily obtained, are presented for PIC151 and PZT5A. With certain resonator geometries, approximate one-dimensional structural models can be used to obtain analytical solutions. Both the method and the program were validated by comparing, for those specific geometries, the results obtained with the one-dimensional structural model and those obtained using our polynomial method. Good agreement was observed.

2. Mathematics and problem formulation

Let us consider a homogeneous solid cylinder of finite height assumed to have undergone a uniform polarization treatment along the thickness direction. R and H are the radius and thickness of the cylinder, respectively. Let us assume that the crystalline z-axis corresponds to the axis of the disc taken as the z-axis of a cylindrical coordinate system $Or\phi z$. Polarization is in the z direction and the disc faces at $z = \pm H/2$ are covered with central electrodes of radius R_0. The electrodes on the top and bottom surfaces are assumed to be very thin and their mechanical properties such as mass and stiffness are assumed negligible. They are connected to a signal generator $V = V_0 e^{j\omega t}$ as shown in Fig. 1.

We assume that constant mass density ρ, elastic moduli at constant electric field $\{C_{ij}\}$, piezoelectric constant $\{e_{ij}\}$, and electric permittivity at constant strain $\{\varepsilon_{ij}\}$ defined with respect to the coordinate axes $Or\phi z$, characterize the elastic and piezoelectric medium of the cylinder. The strain–displacement relations are given by [29]:

$$\begin{aligned} S_{rr} &= \frac{\partial u}{\partial r} & 2S_{\phi z} &= \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \phi} \\ S_{\phi\phi} &= \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \phi}; & 2S_{rz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \\ S_{zz} &= \frac{\partial w}{\partial z} & 2S_{r\phi} &= \frac{1}{r}\frac{\partial u}{\partial \phi} + \frac{\partial v}{\partial r} - \frac{v}{r} \end{aligned} \qquad (1)$$

where u, v, and w are the mechanical displacement components in the radial, circumferential, and axial directions, respectively. We assume the disc material has the symmetry of a hexagonal crystal of the class 6 mm.