Wave Motion 64 (2016) 103–118

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

A probabilistic study of reflection and transmission coefficients of random anisotropic elastic plates

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HIGHLIGHTS

- We analyze wave reflection and transmission through anisotropic elastic plates in the probabilistic framework.
- The effects of material's heterogeneity on reflected and transmitted waves are investigated.
- The highlighting effects of the uncertainty of elasticity properties on the reflection and transmission coefficients are analyzed for different angles of incidence.

ARTICLE INFO

Article history:
Received 16 November 2015
Received in revised form 28 March 2016
Accepted 29 March 2016
Available online 7 April 2016

Keywords:
Random plate
Reflection/transmission coefficients
Semi-analytical finite element method
Cortical bone
Ultrasound

ABSTRACT

This paper presents a probabilistic framework to analyze wave reflection and transmission through anisotropic elastic plates. The elastic plate is sandwiched between two homogeneous fluids and has randomly varied elastic properties in the through-thickness direction. By introducing a stochastic model for quantitative description of heterogeneous elastic properties in the plate, the effects of material heterogeneity on reflected and transmitted waves may be investigated from a probability point of view. The reflection and transmission coefficients are computed *via* the semi-analytical finite element (SAFE) method. A sensitivity study is presented, highlighting effects of the uncertainty of elasticity properties on the reflection and transmission coefficients measured from different angles of incidence.

1. Introduction

A large variety of the natural and artificial materials have unidirectional varying elastic properties. Mantel crust, oceans, composites or bone materials are some of these functionally graded media. For artificial materials, many studies focused on the advantages presented by this type of materials in terms of mechanical behavior. High-tech industries have exploited these properties and developed applications in many fields of the biomaterial and material engineering in the past decades. Consequently, the development of non-destructive evaluation techniques to characterize mechanical behaviors of these materials is a key issue.

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http://dx.doi.org/10.1016/j.wavemoti.2016.03.007

A common setup for ultrasonic testings of this kind of structure consists of an elastic layer sandwiched between two fluids. One of the basic physical concepts for ultrasound quantification of the plate bases on the estimation of reflection and transmission coefficients of ultrasonic waves on its interfaces. While simple and well-documented for testing of isotropic materials, reflection and transmission phenomena for anisotropic and heterogeneous materials are much more complicated from both physical interpretation and technical calculation points of view. General theoretical results on the reflection and transmission phenomena can be found in the comprehensive books of Fedorov [1], Musgrave [2] and Auld [3]. Solutions of the reflection and transmission problem have been studied by Rokhlin et al. [4], Lanceleur et al. [5] for anisotropic elastic plates and by Deschamps and Hosten [6] for orthotropic viscoelastic plates.

The deterministic models assume that the modeling parameters are perfectly known. However, most of the time, only partial information is available on these parameters and the actual values which are obtained *via* experimental measurements. So, it is useful to consider these parameters as uncertain. Among other approaches, probability theory provides an effective and robust framework to take into account such uncertainties. The uncertainty introduced on the parameters allows, in particular, to assess its impact on the interest parameters. In this work, we aim to assess the impact of the anisotropic and heterogeneity of elastic properties of the solid layer on the measured reflection and transmission coefficients. A parametric probabilistic method, which is based on the maximum entropy principle, will be used to generate an optimal probabilistic model by using a minimal parameter number to describe uncertain elasticity field. Moreover, the semi-analytical finite element (SAFE) method will be used to compute the reflection and transmission coefficients of anisotropic heterogeneous solid plates.

In this paper, we are interested mainly on ultrasonic characterization of cortical bone. In this context, some models have been developed by Desceliers et al. [7,8] to evaluate the first arriving velocity in bones by using the axial transmission technique. Here, we will study a three-layer model consisting of a cortical bone solid layer immersed in two fluids. The material properties of the fluids and the bone mass density are supposed to be deterministic. The elasticity tensor of bone tissue is randomly varied in the thickness direction. We investigate the global reflection and transmission coefficients of ultrasonic waves through the cortical bone. A sensitivity analysis of material properties will be performed. In the deterministic model from which the stochastic model is constructed, the cortical bone plate is assumed to be homogeneous and transversely isotropic elastic. Whereas in the stochastic model, the plate is anisotropic and heterogeneous with material properties that vary along the thickness axis. In principle, it will be possible to estimate the physical parameters of bone from the acoustic response. This can be done by solving an inverse problem which will require appropriate resolution strategy. This study presents therefore the first step in the modeling of the direct problem which is indispensable to the identification of material properties.

After this introduction, this paper is organized as follows. In Section 2, the elastoacoustic problem in the solid–fluid coupling is formulated. In Section 3, a numerical approach is proposed for determining the means reflection and transmission coefficients, which are associated with the deterministic model. The probabilistic model of the elasticity tensor and the estimated mean values of these coefficients are presented in Section 4. Then, in Section 5, the validation of the SAFE formulation and the convergence analysis of the proposed stochastic method are presented. Section 6 presents some numerical results simulating a test on a cortical bone specimen immersed in the fluid. Finally, conclusions will be drawn in Section 7.

2. Problem statement

2.1. Description of geometrical configuration

Let $\mathcal{R}(O; \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be the Cartesian reference frame, where O is the origin of the space and $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is the orthonormal basis for this space. The coordinates of a point M are specified by (x_1, x_2, x_3) in $\mathcal{R}$. Fig. 1 presents a two-dimensional geometrical description of the considered problem for which both the incident wave and the mechanical system are assumed to be invariant with respect to x_3. An elastic layer with constant thickness h, which occupies the unbounded domain Ω^b in $\mathbf{e}_1$-axis, is surrounded by two fluid half-spaces Ω_1^f and Ω_2^f. The interfaces between the elastic layer Ω^b and the fluid domains Ω_1^f and Ω_2^f are assumed to be flat and denoted by Γ_1^{bf} and Γ_2^{bf}, respectively, as shown in Fig. 1.

The system is excited by an incident plane and harmonic wave p_I propagating with a pulsation ω in the upper fluid domain Ω_1^f and arriving to the interface Γ_1^{bf} from an angle θ as shown in Fig. 1.

Due to the nature of the source and to the geometrical configuration, the components in the $\mathbf{e}_3$-direction of the displacement vectors in the solid and fluid domains will be zeros and the elastoacoustic wave motion will be independent of x_3. Hence, all derivatives with respect to x_3 vanish in the partial differential equations that govern the wave motion. Consequently, the coordinate x_3 is implicit in the mathematical expressions to follow. The present study is conducted in the plane $(O; \mathbf{e}_1, \mathbf{e}_2)$. These conditions are associated with the plane strain model.

As a result, the domains Ω_1^f, Ω_2^f and Ω^b may be defined by:

$$
\begin{aligned}
\Omega_1^f &= \{M(x_1, x_2); x_2 \geq 0\}, \\
\Omega_2^f &= \{M(x_1, x_2); x_2 \leq -h\}, \\
\Omega^b &= \{M(x_1, x_2); -h \leq x_2 \leq 0\}.
\end{aligned}
\tag{1}
$$