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Check on a nonlocal Maxwellian theory of sound propagation in fluid-saturated rigid-framed porous media

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HIGHLIGHTS

- A nonlocal theory of sound propagation in porous media is validated.
- Porous media are rigid-framed and saturated with a viscothermal fluid.
- The validation geometry is taken to be cylindrical circular tubes.
- Results of nonlocal theory match with those of Kirchhoff's exact solutions.
- Results based on Zwikker and Kosten's local theory are also presented.

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ABSTRACT

A macroscopic nonlocal theory of sound propagation in homogeneous rigid-framed porous media permeated with a viscothermal fluid has been recently proposed in this journal. It accounts for the first time for the full temporal and spatial dispersion effects, independently of the nature of the microgeometry. In this paper this new Maxwellian theory is validated in the case of sound propagation in cylindrical circular tubes, by showing that it matches exactly the long-known direct Kirchhoff–Langevin's solutions.

1. Introduction

In a recent paper [1] a fundamental and new nonlocal macroscopic acoustic theory has been developed, describing in a typically Maxwellian manner the phenomenon of linear sound propagation and attenuation in a macroscopically homogeneous rigid-framed porous medium permeated with a viscothermal fluid, which is governed by the classical Navier–Stokes–Fourier equations of near-equilibrium fluid mechanics [2] at the pore scale. Assuming for simplicity isotropy or propagation along a principal axis, this macroscopic theory introduces a macroscopic acoustic velocity field $\boldsymbol{V} = \langle \boldsymbol{v} \rangle$, where $\langle\ \rangle$ refers to a macroscopic averaging operation, and a macroscopic pressure stress field, denoted H.[1] These two fields appeared to play roles similar to those of the electric and magnetic macroscopic fields, $\boldsymbol{E} = \langle \boldsymbol{e} \rangle$ and $\boldsymbol{H}$, in electromagnetic

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[1] In the absence of the mentioned symmetries, H becomes a symmetric tensor. The theory and procedures described in [1] can be generalized without any difficulty. The permittivities become symmetric tensor operators.

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wave propagation in a medium [3]. The macroscopic pressure field H, in general different from the direct macroscopic average of the pore-scale pressure field, is defined such as the 'Umov' product $H\boldsymbol{V}$ represents the 'acoustic part of the energy current density', by analogy with a similar interpretation of its counterpart in electromagnetics [4], the 'Poynting–Heaviside' product $\boldsymbol{E} \times \boldsymbol{H}$. Gaining in this manner the character of generalized susceptibility functions [5], [1, Section 2.4], the corresponding two acoustic permittivities are determined in terms of two independent action–response problems.

The resulting theory, which must be valid in principle for all microgeometries of the porous medium, and fully takes – for the first time – into account both temporal and spatial dispersion effects, lends itself to direct verifications in various geometries. We give here a check on, in one of the simplest workable test cases: that of the propagation in a medium having parallel cylindrical circular pores of radius R. The possible mode solutions, in the fluid-filled pores, are known since the classical work of Kirchhoff [6] and Langevin [7,8].

We first describe in Section 2, how the general macroscopic acoustic nonlocal theory derived in [1], give in the above simple geometry, definite predictions for the possible wavenumbers and impedances of the axisymmetric modes, irrespective of the frequencies. In Sections 3 and 4, we present the details of the direct Kirchhoff–Langevin's and alternative nonlocal macroscopic Maxwellian computations of these quantities. In Section 5, we show numerically that the two approaches give identical results, irrespective of the frequency range. This provides a clear validation of the new nonlocal theory.

2. General ideas in nonlocal theory, and application to the special case of cylindrical pores

We note that there are usually two different ways to perform the smoothing-out represented by the averaging symbol $\langle\ \rangle$: ensemble averaging or volume averaging, which may be associated to the names of J.W. Gibbs and H.A. Lorentz, respectively. In the Gibbs conception the macroscopic theories do not refer to what happens in a given sample of the medium. They describe what happens, in an averaged sense, in an ensemble of realizations of the medium. The macroscopic mean operation $\langle\ \rangle$ then designate expectation values performed at a given time and spatial position, over the complete set of realizations. In the Lorentz conception, the macroscopic theories do refer to a given sample. The averaging operation $\langle\ \rangle$ is performed by integrating over an averaging volume, at a given time. This volume should be taken sufficiently large to smooth out the irregularities, which requires in general, satisfying some separation conditions on different length scales. Namely, a 'Lorentz' macroscopic wave theory will be well-defined only when the macroscopic medium spatial variations occur at a scale sufficiently large compared to the wavelengths, themselves sufficiently large compared to the volume-averaged scale; for a macroscopically uniform medium the former condition is automatically satisfied and it is sufficient that the wavelengths be large compared to the size of the averaging volume.

We recall now the particular macroscopic acoustic issue we considered in [1], and the way we proposed to solve it. Within Gibbs's conception the whole space is assumed to be divided in two complementary phase regions which depend on the realization $\boldsymbol{\omega}$: the void (pore) region $\mathcal{V}_f(\boldsymbol{\omega})$ which is a connected region fully permeated with a single homogeneous viscothermal fluid, and the complementary solid-phase region $\mathcal{V}_s(\boldsymbol{\omega})$, set to remain motionless and at ambient temperature. The pore-wall region is denoted by $\partial\mathcal{V}(\boldsymbol{\omega})$. Macroscopic homogeneity is assumed.

A small-amplitude acoustic wave disturbance propagates in the fluid in the macroscopic direction x, a principal axis of the medium. At the pore level and for each realization the following classical Navier–Stokes–Fourier linear fluid-mechanics equations can be written in $\mathcal{V}_f(\boldsymbol{\omega})$ (see Eqs. (1) and notations in [1]; $b = \rho'/\rho_0$ is the condensation, τ the excess temperature)

$$\rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p + \eta \nabla^2 \boldsymbol{v} + \left(\zeta + \frac{\eta}{3}\right) \nabla(\nabla \cdot \boldsymbol{v}) \tag{1a}$$

$$\frac{\partial b}{\partial t} + \nabla \cdot \boldsymbol{v} = 0 \tag{1b}$$

$$\gamma \chi_0 p = b + \beta_0 \tau \tag{1c}$$

$$\rho_0 c_p \frac{\partial \tau}{\partial t} = \beta_0 T_0 \frac{\partial p}{\partial t} + \kappa \nabla^2 \tau \tag{1d}$$

with boundary conditions

$$\boldsymbol{v} = 0, \qquad \tau = 0 \tag{2}$$

on $\partial\mathcal{V}(\boldsymbol{\omega})$.

Extending to zero the fields in the solid region $\mathcal{V}_s(\boldsymbol{\omega})$, and introducing the following macroscopic averaged fields ($\boldsymbol{e}_x$ is the unit vector along axis x):

$$V\boldsymbol{e}_x \equiv \langle \boldsymbol{v} \rangle, \quad \text{and} \quad B \equiv \langle b \rangle \tag{3}$$

we argued in [1] that there are 'Maxwell' acoustic density fields $\boldsymbol{D} = D\boldsymbol{e}_x$ and H, and susceptibilities operators $\hat{\rho}$ and $\hat{\chi}$, such that the following macroscopic nonlocal 'Maxwellian' equations hold true:

field equations:

$$\frac{\partial B}{\partial t} = -\frac{\partial V}{\partial x}, \qquad \frac{\partial D}{\partial t} = -\frac{\partial H}{\partial x} \tag{4}$$