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Numerical study on wave propagation in a low-rigidity elastic medium considering the effects of gravity

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HIGHLIGHTS

- We discuss the effects of vertical gravity force on wave propagation in a low-rigidity medium.
- Gravity terms are introduced into the 2D linear FEM for the simulations.
- The fastest phase propagating from a surface point source can be transformed into a gravity wave.
- The fastest phase is identified as a leaking Rayleigh wave.
- The other phases should not have the feature of a gravity wave.

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ABSTRACT

We discuss the effects of vertical gravity force on wave propagation when a material is intermediate between solid and fluid, especially we focus on what kinds of phase are generated and how it propagates on the surface. We introduce gravity terms into the 2D linear finite element method in order to account for the contribution from the gravity. Numerical simulations are conducted for a half-space model and a two-layered, single horizontal layer overlain on a half-space, model. Both models are compared between the results including and excluding the viscosity. The fastest phase propagating from a surface point source, a leaking Rayleigh wave for usual elastic material, is transformed into an interesting phase including some common features to the gravity wave when the gravity effect becomes significant. The viscosity does not affect the fastest phases, whereas it affects other latter phases appearing only for the two-layered model.

1. Introduction

Waves generated by the effects of vertical gravity force propagate on a surface of a fluid medium. They are usually called "gravity waves". The restoring force is proportional to the vertical displacement on the fluid surface. On the other hand, the effects of gravity are not considered in a solid medium because the generated vertical displacement is significantly smaller than that in a fluid medium. The other elastic surface waves such as Rayleigh waves propagating in a solid medium are the major concerns for elastic problems. Both the gravity wave and the Rayleigh wave are well discussed in each field, fluid and elastic theory, respectively.

Several researches have pointed out the relations between the gravity wave and the Rayleigh wave for some particular cases, e.g. low-rigidity soil in the deposit ground, liquefied ground, etc. We first introduce some suggestions and debates about evidences of the gravity wave in a soft ground during actual earthquakes. Lomnitz [1] introduced the experience to see an interesting wave propagating on the ground surface during the 1960 Chile earthquake, and reported that geodetic

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displacement had been observed in spite of a long distance between the epicenter and the observed area. He advocated that it was an attribute of gravity waves excited in the sedimentary basin, and the phenomenon is similar to "seiche" in a confined liquid body. During the 1985 Mexico earthquake, a long-period and long-duration strong ground motion was observed and serious damage occurred in the lake zone of Mexico City, which is a sedimentary basin consisting of very soft clay. Lomnitz [2] suggested that gravity waves were involved in the ground motion, and caused the serious damages. On the other hand, Chávez-García and Bard [3,4] refuted it. They insisted that gravity waves could not explain the long-duration strong ground motion observed in Mexico City because the viscosity of soft soil prevents propagating such a type of wave. For another earthquake, Castanos and Lomnitz [5] introduced the experiences of prograde particle motion by Charles Darwin during the 1835 Chile earthquake. They advocated that the prograde motion is not a particle motion of usual Rayleigh wave.

As the discussions for the actual earthquakes, the evidences have not been completely authorized. Under the present technology, it is still challenge to observe the gravity wave directly even if it exists. This is because that long time records after event triggering up to arrival of the gravity waves, which is slower than the elastic waves, is required. Recently, some seismic networks adopt continuous recordings and wideband velocity seismometers [6]. The advanced observation may be a key to capture the evidences. In addition, the seismometers are required to be located on the soft soil ground because the gravity wave is expected to exist only in such a ground. Unfortunately, few seismic networks target such an observation, and we may need to develop the systems for this objective.

In order to develop the observation systems, we should understand what kinds of phase are generated and how it propagates on the surface in low-rigidity medium. Therefore, the theoretical and numerical discussions about the properties of gravity waves are essential and important. We then focus on the fundamental properties of gravity waves propagating in low-rigidity medium.

Theoretically, the effect of gravity on elastic waves was firstly developed by Bromwich [7] for a simple homogeneous and incompressible media. He introduced the effect, namely, gravity term, into Rayleigh's equation. Gilbert [8] analytically investigated the explicit effects of gravity on wave propagation in a homogeneous half-space and also in a single layer medium with a rigid bottom. Both the materials were incompressible and no viscosities. He found two types of phases propagating in 2D in-plain wave field [9–11]. One is the classical Rayleigh wave and the other corresponds to a wave derived from imaginary roots of Rayleigh's equation. The former was called "locked Rayleigh wave" and the latter "leaking Rayleigh wave". He concluded the results such that (1) the locked Rayleigh wave is dominant when the effect of gravity is absent, (2) the locked Rayleigh wave becomes recessive as the S wave velocity decreases, while the leaking Rayleigh wave, which is dispersive, becomes dominant, and (3) the leaking Rayleigh wave approaches the property of classical gravity wave when the effect of gravity is dominant. Esquivel-Sirvent [12] investigated the transition of gravity waves and surface elastic waves in a half-space consisting of unconsolidated sediment with very small shear velocity. The material was compressible and viscous. He found that a dispersion curve has two branches in low frequencies, one is related to Rayleigh waves and the other is related to gravity waves, and then the response function of the vertical displacement on the free surface has two peaks, one corresponding to Rayleigh waves and the other corresponding to the gravity waves. This suggests that Rayleigh waves can exist together with gravity waves. The effects of gravity on other types of materials and conditions are discussed in Abd-Alla and Ahmed [13], Vinh [14], Ting [15] and etc. Notice that Vinh [14] pointed out that the secular equation in Abd-Alla and Ahmed [13] was not correct due to the incorrect representation of solution.

The debate about the evidences during the 1985 Mexico earthquake was based on the properties of the leaking and locked Rayleigh waves. Lomnitz [2,16,17] suggested a gradual phase transition from leaking Rayleigh waves to gravity waves, based on the results in Gilbert [8] and the observations of long-period and long-duration ground motion. He pointed out that the transition should be discontinuous and nonlinear because surface waves propagate discontinuously and that the stress–strain relationship is nonlinear for the soft soils. However, Chávez-García and Bard [3,4] refuted Lomnitz [2,16]'s suggestion. They insisted that, in a compressible medium with a very large Poisson's ratio, the amplitudes of locked Rayleigh waves are more significant, while leaking Rayleigh waves are small. Lomnitz [18] made the counterargument in regard to these articles by using accelerogram at station SCT recorded as the evidence. He advocated that the phase transition became discontinuous and nonlinear due to the gravity wave. Although Chávez-García and Bard [19] accepted that the discontinuous and nonlinear phase transition were not simulated based on their approach, they pointed out that the existence of prograde Rayleigh waves recorded at SCT does not ensure the existence of gravity waves.

The past studies of wave propagation under the effects of gravity have been based on the theoretical solutions, theoretical-based simple simulations, and/or observed wave records. However, more general conditions, e.g., material heterogeneity, internal energy dispersion, cannot be discussed. In the body of the paper, we develop a numerical method to simulate wave propagation including the effects of gravity, using the 2D finite element method (FEM). FEM is one of the efficient numerical methods to solve a boundary value problem (e.g., Zienkiewicz et al. [20]). We conduct numerical simulations with a variation of gravity accelerations for the homogeneous half-space and two-layered medium, with and without viscosity. Based on the numerical results, we discuss the effects of gravity on wave propagation in a low-rigidity medium.

2. Surface waves in an elastic medium and gravity waves in a fluid medium

We focus on Rayleigh waves in a two-dimensional plane-strain elastic wave field. Let x_1 and x_3 be the axis of the Cartesian coordinates. x_1 axis is directed along the free surface, which represents the horizontal direction. x_3 axis is in the downward