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Interaction of bursts in exponentially graded materials characterized by parametric plots

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HIGHLIGHTS

- Counter-propagation of bursts in exponentially graded materials is studied.
- Oscillations versus material properties are clarified.
- Phase and parametric plots are effectively used for material characterization.
- Special reference case for material inhomogeneity detection is proposed.
- Presented idea is usable for characterization of various inhomogeneous materials.

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ABSTRACT

The nonlinear interaction of tone bursts in functionally graded materials with strongly variable properties is studied resorting to the five constant nonlinear theory of elasticity in the 1D setting. The problem is solved numerically for an exponentially graded material. The influence of the material properties variation on the bursts profiles evolution is traced on the boundaries of the sample. A special case of the bursts interaction by which oscillations evoked by counterpropagating bursts disappear in the homogeneous material is proposed as a reference case for the problem of nondestructive material characterization. The deviation from this special case caused by inhomogeneity in material properties is analyzed by parametric plots. Obtained results may be used by qualitative nondestructive determination of the inhomogeneity in material properties.

1. Introduction

Studies of wave propagation related to innovative functionally graded materials (FGMs) [1] may be regarded as an important component of modern manufacturing due to the need of material inspection [2,3]. Numerous ultrasonic techniques have been developed for this purpose [4]. Continuous ultrasonic waves are used for precise measurements of the amplitude, phase shift and velocity variation of ultrasonic waves as a function of properties of the tested material [5–7]. The pulse and the burst techniques are used to bind the time of flight or the shape disturbances with the material properties [8,9].

The objective of this work is to study the possibility to characterize strongly variable properties of FGMs by the recorded data about the nonlinear propagation and interaction of tone bursts (harmonic waves with a finite length) in the material. The novelty consists in the proposal to use a special case of interaction of bursts by which the oscillations in the homogeneous material disappear, as a reference state for investigations and in the analysis of the influence of material inhomogeneity on the evolution of oscillation profiles using phase plots and their generalizations into parametric plots. The output can be considered as a basis for the bursts interaction technique for the detection of inhomogeneities in material properties.

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At the outset counter-propagation of bursts excited simultaneously on opposite parallel boundaries of a tested material specimen is studied theoretically in the case of the homogeneous physically linear elastic material. The analysis of the analytical solution derived to the one-dimensional equation of motion enables to determine conditions for the special case of superposition of bursts by which oscillations evoked by counterpropagating bursts disappear. This special case is taken as a reference state in further attempts of ultrasonic nondestructive determination of the homogeneity or inhomogeneity in material properties.

Nonlinear interaction of bursts in a physically nonlinear FGM with strongly changing properties is described on the basis of the five constant nonlinear theory of elasticity [10]. The one-dimensional problem is considered and the nonlinear equation of motion is derived. It is difficult to analyze interaction of bursts in FGMs without resorting to some numerical approaches, as the material properties are functions of the coordinates. Here, the nonlinear equation of motion is solved by the program package Maple that uses the finite difference method to obtain the numerical solution.

Interaction of bursts in inhomogeneous material is studied by considering FGMs with exponential variation of the properties close to the boundaries. Such materials are widely employed in many important areas as coatings and interfacial regions for the purpose of reducing residual and thermal stresses and increasing the bounding strength. The profiles of oscillations evoked on the parallel boundaries of the sample of FGM by nonlinear counter-propagation and interaction of bursts are analyzed resorting to the phase plots and their generalization into parametric plots. It is clearly visible that the variation of material properties is distinctly reverberated in the profiles of boundary oscillations. The obtained results may be used as the basis by elaboration of the method for qualitative nondestructive characterization of FGMs with strongly changing continuous properties.

2. Problem formulation

Different effective material gradings for optimal performances of functionally graded materials (FGMs) lead to strong variation of material properties. Deformations of such materials with continuously changing properties are studied here on the basis of the nonlinear theory of elasticity [11]. The wave propagation evoked in the material by external excitation is governed by a second order nonlinear hyperbolic partial differential equation with spatially variable coefficients. The analytical solution to this equation is unknown. The linear governing equation is solved analytically for special cases of material inhomogeneity [12]. The nonlinear problem is solved mainly by approximating the smooth changes of material properties by piling up many homogeneous [13] or inhomogeneous [14] thin layers. Below, the nonlinear one-dimensional wave propagation problem is formulated analytically and solved numerically by the finite difference method making use of the symbolic manipulation software Maple. The one-dimensional governing equation [11] is transformed to a set of three equations

$$
\begin{aligned}
& f(X,t) = V_{,X}(X,t),\\
& g(X,t) = V_{,t}(X,t),\\
& [1 + k_1(X) f(X,t)]\, f_{,X}(X,t) + k_2(X) f(X,t) + k_3(X) f(X,t)^2 = k_4(X)\, g_{,t}(X,t),
\end{aligned}
\tag{1}
$$

where V denotes the displacement, t – the time, X – the Lagrangian rectangular spatial coordinate and the indices after the comma indicate differentiation with respect to coordinate X or time t, respectively. The solution to the set of Eqs. (1) determines directly values of the material displacement and its derivatives with respect to the time and the spatial coordinate.

The second order nonlinear theory of elasticity characterizes the inhomogeneous material by the density $\rho(X)$, the second order elastic coefficients (Lamé coefficients) $\lambda(X)$, $\mu(X)$ and the third order elastic coefficients $\nu_1(X)$, $\nu_2(X)$ and $\nu_3(X)$. In the one-dimensional case the elastic coefficients are grouped to the linear elastic coefficient $\alpha(X)$ and to the nonlinear elastic coefficient $\beta(X)$ in accordance with the equations [15]

$$
\begin{aligned}
& \alpha(X) = \lambda(X) + 2\,\mu(X),\\
& \beta(X) = 2\,[\,\nu_1(X) + \nu_2(X) + \nu_3(X)].
\end{aligned}
\tag{2}
$$

Now, the coefficients of Eq. (1) take the form

$$
\begin{aligned}
& k_0(X) = [\alpha(X)]^{-1},\\
& k_1(X) = 3\,[\,1 + k_0(X)\,\beta(X)],\\
& k_2(X) = k_0(X)\,\alpha_{,X}(X),\\
& k_3(X) = \frac{3}{2}\,k_0(X)\left[\alpha_{,X}(X) + \beta_{,X}(X)\right],\\
& k_4(X) = \rho(X)\,k_0(X).
\end{aligned}
\tag{3}
$$

A large group of FGMs with strongly changing properties are designed for employment as coatings and interfacial regions to improve the resistance of the material to different external affects such as intensive wastage, extreme temperature, etc. [16,17]. In these cases, the variation of material properties may be described by the expression

$$
\gamma(X) = \gamma_0\{1 + \gamma_{11}\exp(-\gamma_{12}X) + \gamma_{21}\exp[\gamma_{22}(X-h)]\},
\tag{4}
$$