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Explicit expression of polarization vector for surface waves, slip waves, Stoneley waves and interfacial slip waves in anisotropic elastic materials

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HIGHLIGHTS

- Explicit expressions of the polarization vectors are given.
- They are given for Rayleigh waves, slip waves and Stoneley waves.
- Unexpected results are obtained for polarization vector for interfacial slip waves.
- It does not depend explicitly on material in the other half-space.
- The special case of orthotropic materials is studied.

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ABSTRACT

We present explicit expression of the polarization vector for surface waves and slip waves in an anisotropic elastic half-space, and Stoneley waves and interfacial slip waves in two dissimilar anisotropic elastic half-spaces. An unexpected result is that, in the case of interfacial slip waves, the polarization vector for the material in the half-space $x_2 \geq 0$ does not depend explicitly on the material property in the half-space $x_2 \leq 0$. It depends on the material property in the half-space $x_2 \leq 0$ implicitly through the interfacial slip wave speed v. The same is true for the polarization vector for the material in the half-space $x_2 \leq 0$.

1. Introduction

Surface waves and slip waves in an anisotropic elastic half-space have been studied [1–3], so have Stoneley waves and interfacial slip waves [4–7]. However, most investigations focus on the secular equation for the wave speed. In this paper we focus our attention on the polarization vector.

In order for the paper to be self-contained we present in Section 2 some basic equations that are needed for the present paper. The polarization vector for surface waves and slip waves in the half-space are studied in Section 3. In Section 4 we consider interfacial waves along the interface of two dissimilar anisotropic elastic half-spaces that are rigidly bonded together. They are known as Stoneley waves [4]. Explicit expression of the polarization vector is presented. In Section 5, we investigate interfacial waves along the interface of two dissimilar anisotropic elastic half-spaces that are in sliding contact.

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It is found that the polarization vector for the material in the half-space $x_2 \geq 0$ does not depend explicitly on the material property in the half-space $x_2 \leq 0$, and the polarization vector for the material in the half-space $x_2 \leq 0$ does not depend explicitly on the material property in the half-space $x_2 \geq 0$. The polarization vectors depend on both materials implicitly through the interfacial slip wave speed υ. This means that, if we change the material in the half-space $x_2 \leq 0$ that does not change the interfacial slip wave speed υ, the polarization vector for the material in the half-space $x_2 \geq 0$ does not change. Likewise, if we change the material in the half-space $x_2 \geq 0$ that does not change the interfacial slip wave speed υ, the polarization vector for the material in the half-space $x_2 \leq 0$ does not change. The polarization vectors are in terms of the components of the surface impedance tensor **M**, which depend on the elastic constants and the wave speed. In Section 6 we present explicit expression of **M** for orthotropic materials.

2. Basic equations

We consider wave propagation in a homogeneous linear anisotropic elastic medium. In a fixed rectangular coordinate system x_i ($i = 1, 2, 3$), the equation of motion is

$$\sigma_{ij,j} = \rho \ddot{u}_i, \tag{2.1}$$

where σ_{ij} is the stress, u_i is the displacement, ρ is mass density, the dot denotes differentiation with time t and a comma denotes differentiation with x_i. The stress–strain relation is

$$\sigma_{ij} = C_{ijks} u_{k,s}, \tag{2.2}$$

$$C_{ijks} = C_{jiks} = C_{ksij} = C_{ijsk}, \tag{2.3}$$

in which C_{ijks} is the elastic stiffness. The C_{ijks} is positive definite and possesses the full symmetry shown in (2.3). The third equality in (2.3) is redundant because the first two imply the third (p. 32 in [8]).

For a two-dimensional steady state motion in the x_1-direction with a constant wave speed $\upsilon > 0$, a general solution for the displacement **u** in (2.1) and (2.2) is [2,3,8,9]

$$\mathbf{u} = \mathbf{a} e^{ikz}, \quad z = x_1 + p x_2 - \upsilon t, \tag{2.4}$$

where $k > 0$ is the real wave number, and p and **a** satisfy the equation

$$[\mathbf{Q} - \rho \upsilon^2 \mathbf{I} + p(\mathbf{R} + \mathbf{R}^T) + p^2 \mathbf{T}]\,\mathbf{a} = \mathbf{0}. \tag{2.5}$$

In the above, the superscript T denotes the transpose, **I** is the identity matrix and

$$Q_{ik} = C_{i1k1}, \qquad R_{ik} = C_{i1k2}, \qquad T_{ik} = C_{i2k2}. \tag{2.6}$$

Introducing the vector

$$\mathbf{b} = (\mathbf{R}^T + p\mathbf{T})\mathbf{a} = -p^{-1}(\mathbf{Q} - \rho\upsilon^2 \mathbf{I} + p\mathbf{R})\mathbf{a}, \tag{2.7}$$

in which the second equality follows from (2.5), the stress computed from (2.2) and (2.4) can be written as

$$\sigma_{i1} = \rho\upsilon^2 u_{i,1} - \phi_{i,2}, \qquad \sigma_{i2} = \phi_{i,1}, \tag{2.8}$$

where the vector

$$\boldsymbol{\phi} = \mathbf{b} e^{ikz} \tag{2.9}$$

is the stress function.

There are six eigenvalues p from (2.5) and six associated eigenvectors **a**. For a steady wave propagating in the half-space $x_2 \geq 0$, p must be complex with a positive imaginary part so that the displacement **u** computed from (2.4) vanishes at $x_2 = \infty$. Let $\hat{\upsilon}$ be the smallest limiting wave speed [3]. For $\upsilon < \hat{\upsilon}$ the six eigenvalues p are all complex. They consist of three pairs of complex conjugates. Let p_n, ($n = 1, 2, 3$) be the eigenvalues with a positive imaginary part and $\mathbf{a}_n$, $\mathbf{b}_n$ be the corresponding **a** and **b** computed from (2.5) and (2.7). The general solution for the displacement **u** and the stress function ϕ obtained from a superposition of (2.4) and (2.9) associated with p_1, p_2, p_3 is

$$\mathbf{u} = \mathbf{A}\left\langle e^{ikz_*}\right\rangle \mathbf{q}, \qquad \phi = \mathbf{B}\left\langle e^{ikz_*}\right\rangle \mathbf{q}, \tag{2.10}$$

$$\mathbf{A} = \left[\mathbf{a}_1, \quad \mathbf{a}_2, \quad \mathbf{a}_3\right], \qquad \mathbf{B} = \left[\mathbf{b}_1, \quad \mathbf{b}_2, \quad \mathbf{b}_3\right], \tag{2.11}$$

$$\left\langle e^{ikz_*}\right\rangle = \mathrm{diag}\left[e^{ikz_1}, \quad e^{ikz_2}, \quad e^{ikz_3}\right], \tag{2.12}$$

$$z_n = x_1 + p_n x_2 - \upsilon t, \quad \upsilon < \hat{\upsilon}, \tag{2.13}$$

and **q** is an arbitrary constant.

Let **U** and **t** be, respectively, the displacement and the surface traction at $x_2 = 0$, i.e.,

$$U_i = u_i(x_1,\ 0,\ t), \qquad t_i = \sigma_{i2}(x_1,\ 0,\ t). \tag{2.14}$$