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Two dimensional self-similar expanding crack problems in elastic half-space

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HIGHLIGHTS

- Exact response of P–SV wave effect has been evaluated using dynamic similarity.
- Transient response has been obtained via body force equivalent.
- Result is valid at least up to the time of arrival of diffracted waves.
- The expression of the surface displacement contains the head wave contribution.
- The spectral behavior of the source time function is moderate than Brune's model.

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ABSTRACT

The application of the property of dynamic similarity is useful to the solution which admits a self-similarity or homogeneous form. One independent variable has been dropped in the present equivalent set of the governing equations. The displacement discontinuity on the crack face and also the displacement field on the surface due to an in-plane shear model over an expanding zone of slippage of arbitrary dip have been obtained. The moving slip edge extends towards the surface with a constant velocity. Cagniard De-Hoop technique has been used here to obtain the two dimensional exact transient response due to the slip in the vertical mode via body force equivalent. The results of the present paper are valid at least up to the time when the diffracted waves from the crack edge have not reached the receiving station. The spectral behavior of the source time function has also been discussed.

1. Introduction

Earthquakes are generally assumed to be produced by slip faulting within the earth and possibly such faulting begins at a point and then spreads radially outward on the fault surface [1]. The crack problems, considered in the present paper, have no characteristic length. This property of dynamic similarity has been proved to be one of the most effective tools and has been used successfully by various authors, both for two or three dimensional cracks (Craggs [2], Burridge and Willis [3], Kostrov [4] etc.). The application of the technique is limited to the solution which admits a self-similarity or homogeneous form and one independent variable can be reduced in the equivalent set of the governing equations. Various formulation for cracks as well as indentation and contact problems have been given by Brock [5,6], Willis [7], Cherepanov and Afanas'ev [8,9], Achenbach and Brock [10] etc. The approach of Cherepanov and Afanas'ev [8,9] is based on the technique of Smirnov–Sobolev's [11–13] functionally invariant solution, making it possible to apply this method to the effective solution of the analogous class of dynamic problems of elastic theory. The techniques of Cherepanov and Afanas'ev [9] and Brock [5] have been

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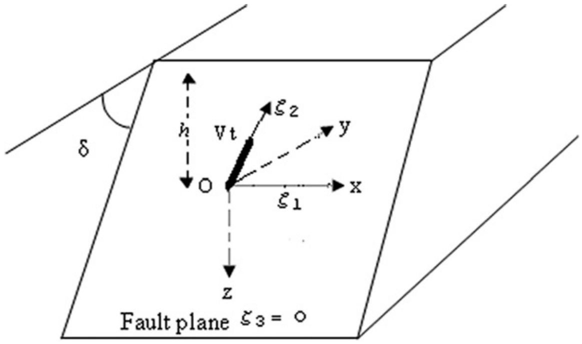

Fig. 1. Expanding crack model under constant shear traction with two coordinate systems.

extended here to obtain the nature of the displacement discontinuity on the crack surface and also the displacement field on the surface due to an in-plane shear stress model over an expanding zone of slippage of arbitrary dip. Achenbach and Brock [10] considered a self-similar sub-surface sliding similar to the present model. They considered the surface effect of the free surface condition on the boundary by taking the reflection from an image source. The reflection from the image source is exact only for the SH-wave. In the present paper, the P–SV response has been considered exactly by considering the surface effect via body force equivalent. The shear stress drop across the fault plane has been considered as linear. It is assumed here that prior to sliding the half-space is motionless and the stress distribution in it is the combination of body stresses due to the weight of the crustal material and tectonic stresses due to the horizontal shear applied at far distances. The resistance to slip obeys coulomb's law of static and kinetic friction. The originality of the present paper lies on the fact that Cagniard De-Hoop technique has been used here to obtain the two dimensional exact transient response, via body force equivalent, up to the time when the diffracted waves from the crack edge have not reached the receiving station. Dynamic similarity has been used here effectively for the determination of the displacement field on the surface.

Georgiadis [14] presented a procedure to analyze the problem of a rapidly propagating crack under a plane-strain general loading with the assumptions of linear elasticity and the method of dynamic similarity has been employed in order to introduce complex-variable methods. A nonstationary axisymmetric problem of model-III elastic wave generation has been considered in an elastic cracked half-space by Skalsky, Stankevych and Serhiyenko [15] through determining the functions of the crack-opening displacements. Nielsen and Carlson [16] demonstrated that when a persistent pulse exists, it expands as it propagates and displays self-similarity akin to the classic crack solution. Richards [17] has suggested that the stress relaxation models of earthquake, involving rupture and shear failure across a planer fault surface, better explain the physics of the faulting process. In the present paper, surface displacement due to the slip in vertical form has been presented and the moving slip edge extends towards the surface along a line from the place of origination, at a depth below the surface, with a constant velocity. The results of the present paper can therefore be used to calculate the theoretical seismogram for such source models at least up to the time when the diffracted waves from crack edge have not reached the station. (See Fig. 1.)

2. Formulation of the problem

We consider an elastic half-space. Let the fault dip at an angle δ, $0 < \delta < \pi/2$ and the dipping fault plane be given by $\zeta_3 = 0$. Let $z = -h$ be the free surface and x-axis be along the strike of the fault plane. The two coordinate systems (x, y, z) and $(\zeta_1, \zeta_2, \zeta_3)$ are related by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & -\sin\delta \\ 0 & \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{pmatrix} = A \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{pmatrix}. \tag{2.1}$$

The half-space is initially at rest ($t \le 0$) under the influence of uniformly distributed tectonic stress (σ_∞) at infinity as well as hydrostatic stress due to the weight of the crustal material. Then at $t \le 0$, the static value of the pressure and in-plane shear stress are given by

$$(\sigma_n)_S = \sigma_\infty \sin^2\delta - \rho g z, \qquad (\tau_v)_S = \sigma_\infty \sin\delta\cos\delta \tag{2.2}$$

where $\rho g z$ is the part due to the weight of the crustal material above the fault surface S, ρ being the mass density and g, the acceleration due to gravity. Also $-z$ is the distance below the free surface. As long as $t \le 0$,

$$|\tau_v| < \gamma_S \sigma_n \tag{2.3}$$

where γ_S is the coefficient of static friction.