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Homogenization of acoustic waves in strongly heterogeneous porous structures

Eduard Rohan *

European Centre of Excellence, NTIS – New Technologies for Information Society, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 22, 30614 Pilsen, Czech Republic

HIGHLIGHTS

- Fluid saturated periodic structures treated by two-scale homogenization.
- Scaling permeability ansatz applied in the dual porosity.
- Homogenized model of pressure waves in rigid skeleton was derived.
- New homogenized frequency-dependent dynamic compressibility involved.
- Dispersion formula obtained, illustrated on layered periodic structures.

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ABSTRACT

We consider acoustic waves in fluid-saturated periodic media with dual porosity. At the mesoscopic level, the fluid motion is governed by the Darcy flow model extended by inertia terms and by the mass conservation equation. In this study, assuming the porous skeleton is rigid, the aim is to distinguish the effects of the strong heterogeneity in the permeability coefficients. Using the asymptotic homogenization method we derive macroscopic equations and obtain the dispersion relationship for harmonic waves. The double porosity gives rise to an extra homogenized coefficient of dynamic compressibility which is not obtained in the upscaled single porosity model. Both the single and double porosity models are compared using an example illustrating wave propagation in layered media.

1. Introduction

By strongly heterogeneous porous structures we mean materials characterized by double porosity—two co-existing systems of connected porosities with large differences in the permeability coefficients. Modeling and analysis of such media became an important topic in the theory of the porous deformable media, see e.g. [1–3]. In the context of the homogenization, it was proposed by Arbogast et al. [4], cf. [5], how the dual porosity can be respected by scaling the permeability coefficients corresponding to the mesoscopic scale.

The acoustic adsorption effects related to interfaces and double porosity and rigid skeleton were studied by Royer et al. in [6] and by Olny and Boutin in [7], starting from the first principles of linearized fluid dynamics at the microscopic level, including thermal effects. Elastic waves in deformable double porosity materials were reported by Berryman and Wang [8]; effects of deformation on the sound absorption were treated by Dazel et al. [9].

* Tel.: +420 377632320.
E-mail address: rohan@kme.zcu.cz.

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In this paper we consider a model which describes fluid motion in the porous medium at the mesoscopic level. At this level the individual fluid-filled pores are not distinguishable so that at any point of the bulk material both the skeleton and fluid phases are present according to the volume fractions. The aim is twofold: Firstly, to derive the two-scale model of waves propagating in such media, so that the effective anisotropic properties can be computed for a specific geometry of the double porosity; Secondly, to compare structure of the derived model with the one of the single porosity model, developed under similar assumptions. Although any detailed analysis of the upscaled models using numerical studies is beyond the scope of this paper, some features are demonstrated on an example of layered periodic media. For some data both the models exhibit quite similar dispersion properties within a small frequency range, however, large discrepancies are encountered in general.

For the sake of simplicity we shall focus only on the waves associated with the fluid motion in a rigid porous skeleton, although deformations of the skeleton could also be accounted for, as it has been done in [10] for the case of single porosity media obeying the system of PDEs proposed by Biot in [11]. Here, assuming zero strains of the porous skeleton, we shall deal with the following reduced model

$$\begin{aligned} &\rho^w \dot{\boldsymbol{w}} + \boldsymbol{K}^{-1}\boldsymbol{w} + \nabla p = \boldsymbol{f}, \\ &\mathrm{div}\boldsymbol{w} + \frac{1}{\mu}\dot{p} = 0, \end{aligned} \tag{1.1}$$

consisting of the Darcy law $(1.1)_1$, relating the total fluid pressure to the seepage velocity $\boldsymbol{w}$ (the effective fluid velocity in the solid skeleton), and the fluid volume conservation $(1.1)_2$. The structure of this model is given in detail in Section 2.1, for comprehension we introduce the notation: by dot we indicate the time derivative, p is the static fluid pressure, ρ^f is the fluid density, density $\rho^w = \phi_0^{-1}\rho^f$ involving the fluid volume fraction ϕ_0 is relevant to the seepage acceleration, $\boldsymbol{K}$ is the permeability, μ is the modulus reflecting compressibility of the fluid. $\boldsymbol{f}$ describes volume forces acting on the fluid phase, thus, being proportional to ϕ_0.

We assume a periodically heterogeneous structure at the "mesoscopic scale"; we consider all the material parameters involved in (1.1) vary periodically with the spatial position and pursue the asymptotic behavior of the model while the frequency of these periodic oscillations grows up to infinity with $\varepsilon \to 0$, where ε is the scale parameter expressing the ratio between the characteristic lengths of the "mesoscopic" and the "macroscopic" scales. To describe the double porosity effects, cf. [4,12], we consider the *dual porosity* where the permeability coefficients are scaled by ε^2. This modeling ansatz can be justified as the result the viscous flow velocity profile and using simple geometrical arguments, see e.g. [13].

The paper is organized, as follows. The model of the fluid-saturated porous medium is introduced in Section 2. In Section 3, the double porous structure is defined in the context of the asymptotic analysis pursued in next paragraphs. The *a priori* estimates are obtained so that the convergence results follow using the periodic unfolding method of homogenization. The homogenized model is then derived using the scale decoupling procedure involving definitions of the local problems for the characteristic response functions. The plane wave propagation is studied in Section 4; in particular, the dispersion models for the single and double porosity media are compared. The two-scale computations are illustrated on an example of the layered medium. Details on the convergence results and some technical proofs are explained in the Appendix which also contains a brief survey of functional spaces.

2. Fluid saturated porous medium

2.1. Model of Darcy flows in porous structure

The model relevant to the mesoscopic scale is based on the concept of the volume fractions of the two phases constituting the saturated medium. By ϕ_0 we denote the *reference* porosity, i.e. the reference fluid volume content. Since the solid phase which forms the porous skeleton is rigid, the fluid velocity[1] $\boldsymbol{v}^f(t,x)$ is directly proportional to the seepage velocity $\boldsymbol{w} = \phi_0\boldsymbol{v}^f$. We consider a compressible fluid; the fluid mass increase represented by $(-\mathrm{div}\boldsymbol{w})$ can is compensated proportionally by the increase of the (static) pore pressure $p(t,x)$,

$$-\mathrm{div}\boldsymbol{w} = \frac{1}{\mu}\dot{p}, \tag{2.1}$$

where μ is called the Biot's modulus; in general, $1/\mu$ describes coupled compressibility of both the phases, however, in our case, due to the rigid solid skeleton, $\mu = k_f/\phi_0$, where k_f is the bulk modulus of the fluid. Eq. (2.1) is supplemented by the Darcy flow model. The generalized Darcy's law reads as

$$\boldsymbol{w} = -\boldsymbol{K}(\nabla p + \rho^f\dot{\boldsymbol{v}}^f - \boldsymbol{f}) = -\boldsymbol{K}(\nabla p + \rho^f\phi_0^{-1}\dot{\boldsymbol{w}} - \boldsymbol{f}), \tag{2.2}$$

where the symmetric second-order tensor $\boldsymbol{K} = (K_{ij}), i,j = 1,\ldots,3$ stands for the anisotropic permeability tensor which is disproportional to the fluid dynamic viscosity. We remark that the Darcy law can be derived by homogenizing the Stokes flow

[1] The mean velocity of the pore fluid.