FISSION AND FUSION INTERACTION PHENOMENA OF THE $(2+1)$-DIMENSIONAL DISPERSIVE LONG WAVE EQUATIONS

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With the aid of symbolic computation, the $(2+1)$-dimensional dispersive long-wave equations are investigated via Bäcklund transformation and the Hirota bilinear method, two types of new multiple soliton solutions with $2n+1$ arbitrary functions for that system are derived. Based on the derived solutions, abundant coherent structures like dromions, compactons and peakons are derived, and moreover, the fission and fusion interactions among different types of localized structures are discussed graphically, which might be helpful for understanding the travel of the shallow water wave.

Keywords: $(2+1)$-dimensional dispersive long wave equations, multiple soliton solutions, fission and fusion interaction, Hirota bilinear method, symbolic computation.

1. Introduction

Nonlinear partial differential equations (NPDEs) are widely used to describe complex phenomena in various fields of sciences. The search of explicit solutions, especially some types of soliton solutions or solitary wave solutions, of NPDEs has attracted certain attention of many mathematicians and physicists. Explicit solutions may help people to better understand the phenomena described by NPDEs. In the past decades, some effective methods to find explicit solutions have been proposed, such as the inverse scattering method [1], Darboux transformation [2, 3], Bäcklund transformation [4], homogeneous balance method [5], the mapping method and extended mapping method [6, 7], the Hirota bilinear method [8–10], etc. Among them, one of the important methods is the Hirota bilinear method, which is an effective direct method to construct explicit solutions of NPDEs.

In this paper, with symbolic computation, we will apply the Hirota bilinear method [9, 10], which is a little different from the usual bilinear method [8], to constructing multiple soliton solutions of NPDEs. As an example, we will study the $(2+1)$-dimensional dispersive long wave equations (DLWEs) [11–37]

$$\begin{aligned} &u_{ty} + v_{xx} + \tfrac{1}{2}(u^2)_{xy} = 0, \\ &v_t + u_x + (uv)_x + u_{xxy} = 0, \end{aligned} \tag{1}$$

where the variable $u = u(x, y, t)$ represents the horizontal velocity of water and $v = v(x, y, t)$ is the deviation height from the equilibrium position of the liquid. Eqs. (1) were first obtained by Boiti et al. [11] as a compatibility condition for a "weak" Lax pair. If we let x be equal to y, Eqs. (1) can be reduced to the (1+1)-dimensional nonlinear DLWEs that describe the travel of the shallow water wave. A good understanding of the solutions for Eqs. (1) is very helpful for coastal and civil engineers to apply the nonlinear water model to coastal harbor design. So far, some efforts have been dedicated to the investigation of Eqs. (1). In [12, 13], some new soliton-like solutions of (1) have been given. In [14–20], some new complexiton solutions, soliton-like solutions and double-like periodic solutions of Eqs. (1) have been derived by means of the generalized algebraic method or the Fan algebraic method with symbolic computation. In [21, 22], the multi solitary wave solution, the rational series solutions and different types of travelling wave solutions for (1) have been derived by the homogeneous balance method and the bifurcation method of planar dynamical systems. In [23–27], some kinds of Jacobi elliptic function solutions of (1) have been given. In [28–35], new variable separation solutions with arbitrary functions of (1) have been obtained by the mapping approach, the extended mapping approach or the extended tanh-function method, the abundant localized coherent structures such as dromion, ring soliton and foldon of (1) have been given, and some interaction phenomena between solitons have been discussed. In [36], the symmetry group theorem of Eqs. (1) has been derived by using the modified CK's direct method, some new exact solutions and conservation laws of Eqs. (1) have been obtained by applying the theorem and the corresponding Lie symmetry. In [37], the Darboux transformations and Lax pair of Eqs. (1) have been determined through the generalized singular manifold method, and the one and two soliton solutions have been obtained by the resulting Darboux transformations. However, as far as we know, multiple soliton solutions, the fusion and fission interaction phenomena of Eqs. (1) have not been extensively studied in the literature.

So, in this paper, we will further study Eqs. (1) by applying Bäcklund transformation and the Hirota bilinear method. The rest of this paper is organized as follows. In Section 2, we will apply the Hirota bilinear method to (1) and construct its multiple soliton solutions. In Section 3, based on the derived solutions, the fission interaction phenomena will be shown graphically. In Section 4, based on the derived solutions, the fusion interaction phenomena are shown graphically. Finally, some conclusions will be made.

2. Two types of multiple soliton solutions of Eqs. (1)

In this section, we apply the Hirota bilinear method [9, 10] to construct multiple soliton solutions to Eqs. (1). From [37], we know that (1) enjoys the Painlevé analysis, the analysis of the leading terms tells that the truncated Painlevé expression of (1) has the following two kinds of Bäcklund transformations:

$$u = \pm 2(\ln f)_x + u_0, \qquad v = 2(\ln f)_{xy} + v_0, \tag{2}$$