SPHERICAL SYMMETRIC DYONIC BLACK HOLES AND VACUUM GEOMETRIES IN $4D$ $N = 1$ SUPERGRAVITY ON KÄHLER–RICCI SOLITON

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In this paper we consider several aspects of dyonic non-supersymmetric black holes in four-dimensional $N = 1$ supergravity coupled to chiral and vector multiplets. The scalar manifold can be considered as a one-parameter family of Kähler manifolds generated by a Kähler–Ricci flow equation. This setup implies that we have a family of dyonic non-supersymmetric black holes deformed with respect to the flow parameter related to the Kähler–Ricci soliton, which possibly controls the nature of black holes, such as their asymptotic and near horizon geometries. We mainly consider two types of black holes, namely a family of dyonic Reissner–Nordström-like black holes and Bertotti–Robinson-like black holes where the scalars are freezing all over spacetime and at the horizon, respectively. In addition, the corresponding vacuum structures for such black holes are also discussed in the context of Morse–(Bott) theory. Finally, we give some simple $\mathbb{C}\mathrm{P}^n$-models whose superpotential and gauge couplings have a linear form.

Keywords: black holes, supergravity, Morse theory, Kähler–Ricci flow.

1. Introduction

Some intensive studies have been done by the author in exploring the nature of solitonic solutions such as domain walls of four-dimensional $N = 1$ supergravity coupled to chiral multiplets [1–4][1]. In those papers, there are considered in particular BPS domain walls of $N = 1$ supergravity in which the scalar manifold can be viewed as a one-parameter family of Kähler manifolds generated by the Kähler–Ricci flow equation [6–8][2]. In other words, we have a chiral $N = 1$ theory on Kähler manifolds whose volume is not fixed and so, the manifold evolves with respect to a real parameter, say τ. This setup implies that the BPS equations, which depend on

[1]Also, we particularly study properties of BPS domain walls from a chiral multiplet in [5].

[2]Some aspects of Kähler–Ricci solitons are shortly reviewed in Appendix B. Alternatively, the interested reader can further consult some excellent reviews of the subject, for example in [9–11].

τ, describe a family of BPS domain walls. Moreover, the flow parameter τ indeed controls the stability of the walls near Lorentz invariant vacua.

In this paper we continue the study by considering another type of solutions, namely black hole solutions which belong to a class of solitonic solutions in four-dimensional $N = 1$ supergravity coupled to chiral and vector multiplets on a Kähler–Ricci soliton, regarded as a one-parameter family of Kähler manifolds. As we will see, the use of Kähler–Ricci solitons in the context of black holes would take several consequences. One of them is that the black hole geometry may change as the flow parameter τ varies. So, we have a collection of black holes with various geometrical properties for each value of τ. We call such an object a family of black holes. In order to obtain such black holes, one has to solve a set of equations of motions such as the Einstein field equation, the gauge field and the scalar field equations of motions by varying the $N = 1$ supergravity action with respect to the metric, gauge fields, and scalar fields together with an additional necessary condition, namely the variation of the flow parameter τ vanishes[3].

Furthermore, the black holes particularly admit a spherical symmetry and are not supersymmetric. Since the vector multiplets are present, the black holes may have both electric and magnetic charges. Such objects are called dyonic black holes. Thus, we have a family of non-supersymmetric, dyonic, and spherical symmetric black holes. Moreover, the analysis of the equations of motions shows that there is an additional potential called black hole potential [12] beside the scalar potential. These potentials play an important role in analyzing the nature of black holes and the corresponding vacuum structures.

In the present paper, we intensely study some aspects of non-supersymmetric dyonic black holes where the scalars become fixed with respect to the spacetime coordinates in some particular regions in spacetime, such as at the horizons, in asymptotic geometries, or all over spacetime. This implies that by solving the scalar field equation of motions there exists a vacuum structure which can be viewed as a collection of critical points of scalar potentials, which is a subset of Kähler manifolds. Such black holes are in general not extreme. The word "extreme" here is coming from some studies of supersymmetric black holes in the context of four-dimensional $N = 2$ supergravity [12–15] and $N = 1$ supergravity [16]. In their case, the black hole is called extreme if its two horizons coincide and on which the black hole potential extremizes at fixed values of scalars. Moreover, in the case of double-extremal black holes the scalars are constant everywhere in the spacetime.

Unlike the domain wall cases in [1–3], since we couple vector multiplets to the theory, we generally have a family of ground states which depend on electric and magnetic charges, and is deformed with respect to the flow parameter τ. The shape of these vacua depends on the type of black holes. They can be characterized by the dimension and the Morse–(Bott) index extracted from the Hessian of the potentials,

[3]We leave some details of this aspect to Appendix C. Moreover, this necessary condition could also be applied in more general cases such as non-supersymmetric theories. But so far it has been considered in the case of BPS-like domain walls in supersymmetric theories [3] .