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Acoustic radiation force and spin torque on a viscoelastic cylinder in a quasi-Gaussian cylindrically-focused beam with arbitrary incidence in a non-viscous fluid

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HIGHLIGHTS

- The axial and transverse acoustic radiation forces on a viscoelastic cylinder are computed.
- The acoustic radiation torque in 2D is derived analytically and calculated numerically.
- The analysis in the off-axial direction is based on Graf's addition theorem.
- Depending on the direction of the shift and absorption, the torque can be either negative or positive.
- Cylinder rotation can be induced in either the clockwise or the anticlockwise direction.

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ABSTRACT

The axial and transverse acoustic radiation force components for a viscoelastic phenolic polymer circular cylinder placed arbitrarily in the field of a zeroth-order quasi-Gaussian focused beam in 2D are computed, based on the partial-wave series expansion (PWSE) method and Graf's addition theorem for the cylindrical wave functions to compute the off-axial beam-shape coefficients. Moreover, the analysis is extended to derive the expression for the acoustic spin radiation torque experienced by the cylinder when the beam is shifted off-axially with respect to the axis of wave propagation. The emergence of the acoustic spin radiation torque experienced by the circular cylinder is a consequence of acoustic attenuation inside its absorptive material. Depending on the direction of the shift, the spin torque is negative or positive, causing the rotation of the cylinder in the polar plane in either the clockwise or the anticlockwise direction, respectively. Moreover, for a fixed position of the cylinder in the off-axial configuration, a spin torque reversal is predicted due to the variation of absorption of both the longitudinal and shear waves. The effect of increasing the surrounding fluid's density is also investigated, which has a substantial effect on the radiation force and spin torque plots. Numerical results are performed with particular emphasis on the properties of the incident beam, the amount of the shift from the center of the cylinder as well as the properties of the surrounding fluid. The analytical formalism allows evaluating the axial and transverse radiation force components and radiation torque for any 2D beam that is generally an exact solution of the Helmholtz equation. The sought applications are in controlled rotation of a particle and its manipulation using "acoustical sheets" (i.e., finite beams in 2D).

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1. Introduction

The controlled rotation of a particle (or multiple particles) has potential applications in manipulating and quantitatively characterizing biological matter, processing materials and the development of mechanical micro-machines [1], especially with the use of surface acoustic wave (SAW) devices [2–4]. The exploitation of the radiation torque inducing the controlled rotation offers a new degree of freedom for the spinning movement of a particle in addition to the translational displacement caused by the acoustic radiation force. A potential immediate benefit is in lab-on-a-chip applications and the development of micro-motors, stirrers or micro-valves in small channels.

In addition to acoustical-based devices, other strategies have used optical [5], magnetic [6–9] or electric [10–12] methods to fulfill adequate rotation functions and angular actuation tasks when a driving approach has an advantage over a known standard of limited operational use. To date, the acoustic radiation force is mainly used to position a particle in space due to the transfer of the linear momentum carried by mechanical waves. Moreover, the resulting acoustic (spin) radiation torque, due to the transfer of (spin) angular momentum, achieves steady rotation of the particle around its center of mass. This mechanical spin rotation is due to the absorption of energy inside the particle's material [13–15], as noted earlier from investigations in optics [16–18].

In acoustics, theories and numerical simulations allowing the evaluation of the acoustic radiation torque exist [15,19–22], which become indispensable tools for experimental design purposes [23–27]. One approach, for example, is to predict under which circumstances how fast an object rotates, or changes its orientation taking into account the effect of the density and characteristics of the surrounding fluid medium. Such variations can have a significant effect on the rotational response of the object and its behavior in the acoustic field. Fast and efficient computations can model virtually any scenario of interest, including limiting cases not entirely attainable using experimental resources. Numerical predictions can be also effective in showing new emergent phenomena related to particle spinning when one cannot perform experiments on unavailable objects or particular cases of interest.

Recent developments and emergent technologies in SAW devices and acousto-fluidics [28] are currently investigating the design and manufacture of an acoustic sensor so as to measure the torque on a rotating fluid droplet (known also as the "SAW torque sensor" [29]). The production of such a device in the SAW-microfluidics context may enable a simple and reliable on-chip non-contact measurement and estimation of the angular velocity for potential characterization of the droplet's physical properties.

Although earlier works considered the analytical development for the radiation torque theory in 3D [15], an extended formalism for the case of 2D-*beams* is necessary, since SAW ultrasonic devices and transducers often deal with "acoustical sheets" (i.e., finite beams in 2D) in most applications, often using a focused wavefront [30]. The aim of this work is therefore directed towards this goal to develop a complete theoretical and computational analysis for the radiation force and torque in 2D, where the latter seems to be non-existent yet. The theoretical development is based on the partial-wave series expansion (PWSE) method in cylindrical coordinates and the description of the incident zeroth-order quasi-Gaussian focused beam via the so-called beam-shape coefficients (BSCs). The case of a sound absorptive phenolic polymer cylinder submerged in non-viscous water and placed arbitrarily with respect to the incident beam is considered. The axial and transverse radiation force components are derived and evaluated along with the radiation spin torque component using the far-field scattering method which does not induce any approximation in a non-viscous fluid. Computations emphasize the effect of shifting the cylinder in 2D off the beam's axis, including the influence of the focusing properties of the illuminating beam. The present investigation is of noble interest essentially due to its inherent value as a canonical problem in physical acoustics. It provides a rigorous and reliable method for analyzing the effects of the acoustic radiation force and torque on a (viscoelastic) cylinder. Notice also that there have been recent advances in the evaluation of the scattering [31,32], radiation forces [32–34] and torques [34] for 2D elliptical cylinders. Promising applications include the design and optimization of SAW-based microfluidics devices and other particle manipulation applications using integrated devices capable of performing a complete conventional laboratory process, from start to finish.

2. Theory

The analysis to derive the acoustic radiation force and torque expressions is based on integrating the radiation stress tensor [35] and its moment [19] over the surface of the object (or another virtual surface enclosing it), which is expressed in terms of the total (incident + scattered) first-order velocity and pressure fields. Therefore, the scattering off the cylinder is considered first, which will be subsequently used in the evaluation of the force and torque.

Consider a two-dimensional (2D) zeroth-order quasi-Gaussian focused beam propagating in a nonviscous fluid, and incident with an arbitrary direction upon a circular cylinder as shown in Fig. 1. It proves convenient to use the far-field expressions for the incident and scattered velocity potential fields in order to derive the analytical formulas for the force and torque experienced by the cylinder, since the surrounding fluid medium is non-viscous. It has already been established that the derivation for the force [36–39] and torque [13,15,19] based on the far-field scattering approach provides identical results obtained from the near-field scattering as long as non-viscous fluids are concerned.

Thus, the farfield expressions for the incident and scattered acoustic velocity potential fields are expressed, respectively, as [40]