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Wave motion in a thick cylindrical rod undergoing longitudinal impact

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HIGHLIGHTS

- Longitudinal impact of thick elastic rods solved without the Skalak's decomposition.
- Von Schmidt waves are crucial for stress peaks at the rod axis and its vicinity.
- The first longitudinal mode is sufficient to detect the wavefront of Rayleigh wave.
- For long times the 1D theory approximates 3D solution near the place of impact.
- Presented results give a deeper insight into the wave processes induced by impact.

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ABSTRACT

The paper presents a new formulation and the comprehensive analytical solution to longitudinal impact of thick elastic rods. In contrast to previously published works, the solution is derived based on the exact three-dimensional theory without using the classic Skalak's decomposition. This new direct approach makes the analytical solution more transparent and much easier to obtain. The resulting formulas for basic mechanical quantities are derived using the residue theorem and their evaluation is made in such a way that the accuracy of presented results is significantly higher than those previously published. Based on these results, the transient wave phenomena occurring in the rods are discussed in detail. Additionally, the solution in time domain is obtained also by semi-analytical approach making use of numerical inverse Laplace transform. It is shown that the selected FFT based algorithm is accurate and robust enough, such that the analysis of wave motion in spatial and time domain can be done effectively preserving the results precision. Presented solution can be used as a benchmark for verification of numerical and experimental methods applied to elastodynamics problems.

1. Introduction

It is a well known fact that every loading of a solid is a dynamic process involving the propagation and reflection of waves. Stress waves are often generated by the mechanical impact of solids. One of the simplest impact situations is the wave motion in thin rods due to longitudinal impact loading. A complete treatment of one-dimensional (1D) longitudinal vibrations caused by impact on rods was first provided by St.-Venant in 1883 [1]. The longitudinal collinear impact of two

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thin rods (considered in the sense of the elementary 1D theory) is a technically important problem, since it gives the basis for many experimental studies of wave propagation, see [2,3]. If the striker rod and the impact rod are of the same elastic material and cross-section, the 1D rod theory predicts a rectangular pulse that propagates along the impacted rod without any distortion. In practice, imperfections in the physical system and inadequacies of the governing 1D theory will cause deviations of the generated pulse from the ideal rectangular shape. It turns out that the elementary 1D theory is true only for long-wave components of the pulse and it cannot be used to describe the propagation of pulses with a steep wavefront. A certain improvement of the elementary 1D theory leads to the Love theory which considers inertial effects in the radial direction of a rod [3]. The Love theory extends the frequency region of applicability of the 1D theory, however, the physics is still too simple to cover the high frequency problems. In spite of this imperfection, the "one-mode" Love theory is often successfully applied to long waves and far fields, instead of the more complex "two-mode" approximate Mindlin–Herrmann theory or "three-mode" Mindlin–McNiven theory, see e.g. [4].

The theory of the longitudinal wave motion in thick cylindrical rods, based on three-dimensional (3D) elastodynamics, was independently proposed by Pochhammer (1876) [5] and Chree (1889) [6]. Due to the inability of simultaneous fulfillment of the boundary conditions on the sides and at the ends of the cylinders, the exact solution of the finite rod problem remains unsolved yet [7]. Therefore, all rigorous analytical solutions to 3D problems which are known to us discuss the semi-infinite or infinite rods.

The first 3D study on longitudinal impact of two semi-infinite rods was considered by Skalak [8]. His approach consists in the superposition of two solutions. The first involves the impact of two semi-infinite rods which are completely constrained against lateral motion. It leads to the known solution of half-spaces collision. The second solution lies in solving the problem of an infinite rod, initially at rest and unstressed, but subjected to an equal and opposite radial stress traveling along the surface of the rod. Due to the superposition principle, the summation of these both solutions produces the traction-free lateral surfaces of the two colliding semi-infinite rods. This approach is sometimes called the "Skalak's decomposition". Among other papers published on this topic at that time we mention the paper by Folk et al. [9], which describes the wave motion due to a suddenly applied stress at a terminal face of a semi-infinite bar. From papers appeared latter we adduce the Motoyama solution for longitudinal impact of bars with an elliptical cross-section [10] or Tanaka solution for longitudinal impact of bars with a rectangular cross-section [11].

The solutions presented in the works aforementioned are based on the application of the classic method of integral transforms. It means that the resulting formulas for studied mechanical quantities have the form of infinite series of improper integrals. Due to the limited computing resources, the improper integrals could be only expressed by some approximation method, e.g. by the saddle point method as done also in the Skalak's paper [8]. Such analytical results are then valid only for long times and long distances from the place of impact. An indication of the Skalak's problem solution for short times was first published by Spillers and Callegari in 1969 [12]. Only in the year 1996, Vales et al. [13] completed the exact analytical solution started by Skalak and presented its extension to the near field with extensive numerical calculations. With respect to the limited computing power at that time, only 50 longitudinal modes and a rather narrow interval of frequencies could be taken into account when the response in time domain was analyzed. To complete the survey of analytical works on this subject, one could also mention two theoretical papers by Malkov [14,15]. In the first paper, the behavior of the dilatation and the only component of the rotation vector in the vicinity of wavefronts due to the longitudinal impact of semi-infinite plane elastic bars are studied using the Sobolev–Smirnov method. The second paper describes the behavior of reflected waves. Both papers unfortunately contain no diagrams which could elucidate the presented results.

Due to the limits of modes and frequencies mentioned above, the results presented in [13] are burdened with spurious oscillations that are in essence similar to those observed when a finite number of terms of infinite series is considered. In spite of these inaccuracies, however, the longitudinal stress waveforms shown in [13] have been used in the recent past as a benchmark solution in a number of works, e.g. [16–21]. The authors of mentioned works used this solution for the validation of the finite element method applied to linear problems of elastodynamics and to non-linear contact problems.

In view of this ongoing interest in the analytical solution of longitudinal impact of thick rods, this paper focuses on a new formulation of the problem and delivers analytical results of higher accuracy than previously published in literature. In particular, a new direct formulation without using the Skalak's decomposition and the final analytical formulas for selected mechanical quantities are presented. The results in a form of dimensionless axial stress are then used for a detailed description of wave phenomena occurring in the rods and for a discussion concerning the accuracy of obtained results in comparison with those presented in [13]. With respect to the fact that the exact Laplace inversion based on the Cauchy's residue theorem as used in the first part of this paper is very demanding process, mainly in the sense of CPU time, numerical approach to the Laplace inversion is adopted in this work, as well. The accuracy and effectiveness of the used FFT based algorithm is discussed at the end of this paper.

2. Problem formulation

The problem of longitudinal impact of thick elastic cylindrical rods of diameters $2a$ is considered. This problem is understood to mean the collision of two semi-infinite thick cylindrical rods which move coaxially against each other at identical velocities v_0. All the points of the plane faces of the rods are assumed to meet at the same instant $t = 0$ when they are suddenly stopped. With respect to the geometry of solids studied a cylindrical coordinate system r, ϑ, z is assumed. Due to the rotational symmetry problem, the radial, axial and tangential components of displacement are