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Surface wave interaction with rigid plates lying on water

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HIGHLIGHTS

- A new integral equation formulation and solution to solving problems involving thin plates lying on water.
- Simple and numerically efficient computations.
- New results for rectangular and parallelogram-shaped plates and eigen-values for sloshing in ice-holes.

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ABSTRACT

In this paper, a variety of problems concerned with the interaction of water waves with fixed horizontal plates lying on the surface of a fluid are investigated.

Firstly, solutions are presented to the problem of the scattering of incident waves by: (i) infinitely-long plates of constant finite width (often referred to as the two-dimensional 'finite dock problem') and (ii) finite plates which are either rectangular or parallelogram-shaped. Secondly, hydrodynamic coefficients due to forced motions of plates are also considered. Finally, eigenvalue problems associated with free oscillations of the surface in long channels of uniform width and finite rectangular holes in an otherwise infinite rigid plate covering the surface are considered.

A common method of solution is applied to all problems which involves using Fourier transforms to derive integral equations for unknown potentials over finite regions of space occupied by either plates or the free surface. Integral equations are converted, using the Galerkin method, into second-kind infinite systems of algebraic equations. In each problem numerical approximations to the solutions are found to converge rapidly with increasing truncation size of the infinite system making this approach both numerically efficient and accurate. Some comparisons with existing results are made, and new results for finite plates are demonstrated.

1. Introduction

The reflection and transmission of surface gravity waves by a rigid plate or 'dock' on the surface of a fluid is a classical problem in the study of linearised water waves. For example, when the plate covers the half-plane – the so-called semi-infinite dock problem – an explicit expression for the reflection coefficient can be found using the Wiener–Hopf technique ([1–3] for example). For a plate that is infinitely-long in one direction and of uniform constant width in the perpendicular direction – the so-called 'finite dock problem' – exact solutions are no longer possible and various techniques have been employed all leading to approximations of the reflection and transmission coefficients. See, for example, [3,4] who base solutions on short wave asymptotic approximations and [5,6] who use domain decomposition in finite water depth combined with a modified residue calculus technique, and references therein.

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We start this paper by revisiting the finite dock problem in fluid of infinite depth. The detailed changes required for constant finite depth are easily implemented following the methods we employ (or consult [7]) but are omitted here in order to retain simplicity. Owing to the geometry, the problem under consideration is quasi two-dimensional, the obliqueness of the plane monochromatic incident waves implying that the governing equation is transformed from Laplace's to the modified Helmholtz equation. Investigation of the oblique finite dock problem allows us to establish the basic method of solution and assess the performance of the numerical method against known results. The solution is derived by initially using Fourier transforms to formulate an integral equation for the unknown potential under the plate. This is expanded in a series of Legendre polynomials and the integral equation is thus transformed into a second-kind infinite system of equations for the coefficients in the expansion. This is shown to involve matrix elements which are easy to compute accurately. Furthermore, results show that the system of equations converges rapidly with increasing truncation size. This general approach forms the basis of recent work on similar problems involving submerged plates by [7] and has similarities to work by [8] for radiation of internal gravity waves by discs oscillating in a stratified fluid and [9] for scattering of surface waves by cracks in elastic solids.

The application of the approach adopted here appears new. Unlike modified residue calculus methods (e.g. [6]) the method can be applied in water of infinite depth and finite depth. For finite depth, knowledge of the all roots of dispersion relations in the complex plane are not necessary – this is not an issue in the problems illustrated here, but could be if the method were applied to problems with more complex boundary conditions such as those involving elastic or porous plates. The general approach is restricted to those problems for which taking Fourier transforms is possible but has the advantage over approaches based on Green's functions of bypassing technical difficulties with singularities (see [10] for example). From a practical perspective the numerical implementation of the solution is very simple and for low frequency/short plates results in rapidly convergent numerical solutions requiring little computational effort. For high frequency/longer plates, the numerical method requires more effort as it aims to reconstruct properties of the field along the plate in a finite series of functions. In these cases, methods which incorporate the explicit solutions to the solution of a semi-infinite dock (see [6]) have an advantage.

One major factor in adopting this method of solution is that it lends itself to being extended to a more general class of problem and this is pursued throughout the remainder of the paper. Thus, we continue the paper by considering the three-dimensional scattering of waves by a thin finite plate of arbitrary shape fixed in the free surface. Apart from where the plate is circular (when separation of variables can be used, e.g. [11]) there appears to be little work on this problem. For floating elastic plates of arbitrary planform [12,13] have derived a semi-analytical approach based on Green's function combined with a boundary element discretisation. For certain geometrically simple shapes of plate shown in this paper (rectangles, circles, parallelograms) the Fourier transform approach avoids this level of complication.

To extend the approach used for the finite dock problem to this new problem we simply implement a double Fourier transform to develop integral equations which retain the same overall structure of the solution as before. The reduction to an infinite algebraic system of equations now involves terms which are more computationally intensive, being defined by double integrals over Fourier space as opposed to single integrals previously. However, since the numerical method converges rapidly with increasing truncation size, numerical solutions remain quick to compute. Another advantage of the method described in the paper is that solutions are easily adapted for parallelogram-shaped plates.

In the last main section, we consider a set of problems in which the regions occupied by the free surface and the plate are interchanged. Thus the fluid is now bounded above by an infinite rigid lid within which a finite hole is cut to leave the fluid exposed to the atmosphere. The problem is one of determining the natural sloshing modes in this so-called 'ice fishing problem'; see [14]. Again, this is a problem with a long history particularly in two dimensions; papers of particular note in this respect include [15–17].

2. The oblique-incidence finite-dock problem

Cartesian coordinates are used with $z = 0$ coinciding with the mean free surface and the fluid extending into $z < 0$. A rigid horizontal plate is placed on the surface, $z = 0$, and extends uniformly in the y direction and from $x = -a$ to $x = a$ in the x-direction. Assuming time-harmonic incident waves of angular frequency ω making an angle $\theta_0 \in (-\frac{1}{2}\pi, \frac{1}{2}\pi)$ with respect to the positive x direction, the velocity field can be found from the gradients of $\Re\{\Phi(x, y, z)e^{-i\omega t}\}$ where the velocity potential $\Phi(x, y, z)$ satisfies

$$\nabla^2 \Phi = 0, \qquad z < 0 \tag{1}$$

the linearised free surface condition

$$\left(\frac{\partial}{\partial z} - K\right)\Phi = 0, \quad z = 0 \tag{2}$$

where $K = \omega^2/g$ and $g = 9.81\ \mathrm{ms}^{-2}$ is gravity, and

$$|\nabla \Phi| \to 0, \qquad z \to -\infty. \tag{3}$$