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Analytical extension of Finite Element solution for computing the nonlinear far field of ultrasonic waves scattered by a closed crack

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HIGHLIGHTS

- Computation of nonlinear scattering by closed cracks.
- Coupling of Finite Elements and analytical propagation.
- Crack modeled by unilateral contact with Coulomb's friction.
- Application to higher harmonics generation and non-collinear mixing of shear waves.

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ABSTRACT

Nonlinear scattering of ultrasonic waves by closed cracks subject to contact acoustic nonlinearity (CAN) is determined using a 2D Finite Element (FE) coupled with an analytical approach. The FE model, which includes unilateral contact with Coulomb friction to account for contact between crack faces, provides the near-field solution for the interaction between in-plane elastic waves and a crack of different orientations. The numerical solution is then analytically extended in the far-field based on a frequency domain near-to-far field transformation technique, yielding directivity patterns for all linear and nonlinear components of the scattered waves. The proposed method is demonstrated by application to two nonlinear acoustic problems in the case of tone-burst excitations: first, the scattering of higher harmonics resulting from the interaction with a closed crack of various orientations, and second, the scattering of the longitudinal wave resulting from the nonlinear interaction between two shear waves and a closed crack. The analysis of the directivity patterns enables us to identify the characteristics of the nonlinear scattering from a closed crack, which provides essential understanding in order to optimize and apply nonlinear acoustic NDT methods.

1. Introduction

The detection of damage at early stage of fracture is of primary importance in many technologies, such as nuclear power plants or aeronautics. In the case of micro-cracks or closed cracks, the linear ultrasonic methods are less efficient but it has

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been proven in the past decade that nonlinear ultrasonics can bring an answer to this challenge. Many of these methods are based on the enrichment of the frequency content of the probing waves when interacting with the damage. The nonlinear effect involved in this interaction is related to contact dynamics and is called Contact Acoustic Nonlinearity (CAN) [1]. Sub- and higher-harmonic generation [1], frequency-modulation [2] have been shown to be sensitive to micro-cracks or closed cracks. For an overview of nonlinear acoustics applications see [3–5].

Numerically, both longitudinal and shear wave propagation through a rough surface were investigated in [6] using an interface contact model based on Hertz theory, for time harmonic incidence. The partial contact model was subsequently applied to model scattering from surface breaking cracks [7], and numerical simulations indicated efficient production of second harmonics. Another approach consists in introducing interface stiffness to account for quantitative transmission and reflection wave and harmonic generation [6,8,9]. Preisach-Mayergoyz space representation has been used to model CAN including hysteresis effect [10,11], and has also been implemented in time domain Finite Element (FE) model [12]. CAN has also been modeled by unilateral contact law with Coulomb's friction in time explicit Boundary Element Method (BEM) [13–15] or FE model [16]. In [13], the BEM was used for the study of SH slip motion on an arbitrary interface, which has later been extended to study of the interaction between a P wave or a SV wave with a pre-open or pre-stressed crack under normal incidence [14,15]. *P* waves correspond to longitudinal waves, whereas SH and SV waves correspond respectively to out-of-plane shear waves and in-plane shear waves. It was shown in [14,15] that the amplitudes of the higher harmonics of the scattered far-fields can be useful in determining both the pre-stress and the frictional coefficient. The same model was also used to treat the interaction between a crack and a SV wave for a given angle of incidence [15]. In this case, the crack was initially closed, and free of any pre-stress. The solution for the far field was obtained but the frequency content was not analyzed. It appears that considerable efforts have been made in previous work in order to model CAN but that the nonlinear far-field components have been only partially computed and that the nonlinear scattering patterns were not determined.

The purpose of this paper is to propose a generic method to compute the far field solution for the waves scattered by a closed crack when CAN is activated, including converted modes. In particular, the method is developed to provide a solution for the new frequency components generated by the contact dynamics triggered at the crack by the incident wave. Actually, for the development and the application of non-destructive methods based on nonlinear acoustics, it is fundamental to capture these new frequency components. Thus, the study scattering patterns corresponding to scattered higher harmonics is a valuable knowledge. Because no analytical solution is available for the contact dynamics problems, the first step of the method consists in using a 2D FE model to compute the near field solution in the time domain. The crack can be closed by a pre-stress and is modeled by a unilateral contact law with Coulomb friction. Once the solution for the scattered waves in the near field is obtained numerically, it is converted to the frequency domain and then extended in the far field domain using an analytical method. This second step uses Hankel's functions to compute the far field solution, which allows us to plot the directivity patterns of the scattered wave for different frequencies. This two step approach is similar to the one used by Hunt et al. to compute the linear field radiated from elastic structures in a fluid domain [17,18].

The paper is organized as follows. The two step method is described in Section 2, with first a description of the FE model, and second the analytical model used to propagate the solution in the far field. In Section 3, the method is applied to compute the directivity patterns for two examples of nonlinear acoustic problems. The first case is the nonlinear scattering of a wave by a closed crack of different orientations. The directivity patterns of the higher harmonics are obtained. The second case deals with the scattering of a longitudinal wave resulting from the interaction between two incident shear waves and a closed crack. This last example corresponds to an application of the non-collinear mixing method [19–22].

2. Computation of the nonlinear far field scattered waves

A two step procedure is proposed for computing the scattered far field resulting from the nonlinear interaction between one or two waves and a source of nonlinearity such as a closed crack. In this particular case, the contact dynamics generates the nonlinearity and therefore is the source of the higher harmonics. The proposed methodology allows us to plot the directivity patterns corresponding to the new generated frequency components. Longitudinal and shear waves can be scattered by the crack and the two modes are considered here.

The first step consists in using a FE model to treat the nonlinear interaction between an incident acoustic wave and a closed crack of finite extent, taking into account the contact dynamics. The FE model is solved using the code Plast2 [23]. The size of the FE model being limited in space due to computational cost, the numerical solution regarding scattered waves is restricted to the near field (a few wavelength away from the crack). To obtain the directivity pattern of the scattered waves and to gain understanding of the nonlinear scattering, the solution must be computed in the far field. The second step consists in extending the numerical solution in the far field based on analytical expressions. This is done for both longitudinal and shear scattered waves.

2.1. Finite element model for the near field solution

2.1.1. Problem statement and generic FE model

This section aims to describe the FE models used to treat the interaction between waves and a closed crack, including nonlinear effects due to the CAN. The description of the model is generic, giving the modeling principles, with some specific