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Lamb wave propagation in a plate with step discontinuities

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HIGHLIGHTS

- Lamb wave-scattering at step discontinuities.
- Least-square solution by employing analytical expressions.
- Inclusion of nonpropagating and evanescent modes in near-field solutions.
- Good agreement between analytical and numerical findings.

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ABSTRACT

Ultrasonic Lamb waves are affected by structural discontinuities (e.g. stiffeners, ribs or joints), and it is necessary to understand the quantitative characteristics of their influence on the waves so that the structural features are not misidentified as defects in nondestructive evaluation (NDE) applications. In this paper, the benchmark problem of Lamb wave reflection at a free end as well as the interaction of Lamb waves with upward and downward step discontinuities is solved using an analytical approach. The free-end problem is considered to show the influence of the decaying waves on the power flow in the plate. In all problems, a near-field solution is obtained, and power flows past the cross-sections of the plate are evaluated. Furthermore, amplitude spectra and results for different step sizes are presented. The analytically derived results are compared with those from transient finite element simulations, showing good agreement.

1. Introduction

Ultrasonic guided waves can be used for inspecting large plate-like structural components due to the fact that these waves can travel long distances, and their propagation speed and amplitude are strongly influenced by the presence of defects in their propagation path. A careful analysis of the waveforms can, in principle, lead to fast and reliable damage detection and characterization. However, the waves are also affected by structural discontinuities (e.g. stiffeners, ribs or joints), and it is necessary to understand the quantitative characteristics of their influence on the waves so that the structural features are not misidentified as defects in nondestructive evaluation (NDE) and structural health monitoring (SHM) applications.

Wave propagation in the far-field of structural discontinuities in isotropic plates has been studied by numerous authors, employing a variety of methods. Early work provided approximate solutions of the Lamb wave reflection problem from the free end of a plate using analytical methods [1–3]. Other problems involving geometrical discontinuities were studied using finite element (FE) simulations [4,5], semi-analytical FE methods [6–9], or semi-analytical boundary element (BE)

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methods [10–12]. Analytical near-field solutions have been derived for different through-thickness and flat-bottomed discontinuities [13–15]. Theoretical and numerical models of the free-end problem were also validated by ultrasonic experiments [16,17]. Mal and Knopoff [18] carried out detailed studies of the transmission and reflection of Rayleigh waves at a step on the surface of a half-space, and at the corner of a wedge [19] using a Green's function approach. However, to the authors' knowledge, theoretical studies on Lamb wave propagation in plates containing step-like discontinuities are rather sparse. A related problem involving Lamb wave propagation in a lap-jointed plate was investigated by Chang and Mal [20], using a semi-analytical approach in which the domain including the discontinuous region is modeled by finite elements and the infinite domain external to it is modeled by propagating Lamb wave modes. It was shown that the nonpropagating and evanescent modes become negligibly small at a distance of about two plate thicknesses from the joints. This is consistent with the conclusion reached by Vasudevan and Mal [21] from a detailed investigation of the propagation of Lamb waves in a plate from internal and surface sources.

In this paper, Lamb wave propagation in the near- and far-fields of upward and downward step discontinuities at the surface of a plate is studied. This problem is of particular interest for lightweight aeronautical and aerospace composite structural components that are stiffened by stringers or ribs, forming an I- or a T-shaped discontinuity. Depending on the properties of the generated ultrasonic pulse (frequency, number of cycles, wavelength), such stiffeners can be treated as semi-infinite structures, or as a finite-length discontinuity. Referring to previous experiments [22], it was found that the short pulse at a typical excitation frequency for NDE purposes is not yet reflected back from the trailing edge of a flat stiffener while the incident wave interacts with the leading edge. Hence, in this paper, an analytical framework that deals with the scattering of Lamb waves at upward and downward steps is developed. Based on the completeness of the set of Lamb waves [23], an approximate analytical solution is obtained in which the boundary and continuity conditions at the discontinuous cross-section are satisfied in a least-square sense. A similar approach has been used by Castings et al. [24] to study the interaction of Lamb waves with cracks that are perpendicular to the propagation direction of the waves, as well as by Santhanam and Demirli [25] to investigate the reflection of an obliquely incident Lamb wave from a free end. Findings from the analytical solution for the step discontinuities are compared to those from transient FE simulations. The method is also applied to the free-end problem, and the results are compared with those obtained in earlier studies [1–3,20] in order to validate the accuracy of the method.

The outline of this paper is as follows. In Section 2, a brief description of the roots of the Lamb wave dispersion equations is given, and all physically relevant modes are identified. In Section 3, an analytical technique is formulated in the near-field of discontinuities. In Section 3.1, a near-field solution of the reflection of an incident Lamb wave at the free end of the plate is derived, and the results are compared with those available in the literature. In Sections 3.2 and 3.3, least-square solutions for the scattered waves in the near- and far-field of upward and downward step discontinuities are determined. Results from the analytical solution for these problems are compared with those from numerical simulations in Section 4. Concluding remarks are presented in Section 5.

2. Propagating, nonpropagating and evanescent Lamb waves

In this paper it is assumed that the material of the plate is isotropic and perfectly elastic, occupying the domain $-\infty < x < \infty, -H < y < H, -\infty < z < \infty$ (see Fig. 1). Assuming a state of plane strain, the displacement and stress components $u_i(x, y, t)$ and $\sigma_{ij}(x, y, t)$ within the plate can be expressed in the forms

$$u_i(x, y, t) = \frac{1}{\pi} \operatorname{real} \left(\int_0^\infty \bar{u}_i(x, y, \omega)\, e^{i\omega t}\, d\omega \right) \tag{1}$$

$$\sigma_{ij}(x, y, t) = \frac{1}{\pi} \operatorname{real} \left(\int_0^\infty \bar{\sigma}_{ij}(x, y, \omega)\, e^{i\omega t}\, d\omega \right), \tag{2}$$

where $i = 1, 2$, and $\bar{u}_i$ and $\bar{\sigma}_{ij}$ are the Fourier time transforms of the displacement and stress components. Introducing a second Fourier transform to $\bar{u}_i$ and $\bar{\sigma}_{ij}$ with respect to x, they can be expressed in the forms

$$\bar{u}_i(x, y, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{u}_i(k, y, \omega)\, e^{ikx}\, dk \tag{3}$$

$$\bar{\sigma}_{ij}(x, y, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{\sigma}_{ij}(k, y, \omega)\, e^{ikx}\, dk. \tag{4}$$

The integrand in Eqs. (3) and (4) depends on the thickness and material properties of the plate in addition to y and the two transform parameters ω and k. The integral is known as the wavenumber integral [26] due to the fact that the integration variable k can be thought of as the wavenumber of a wave propagating with the speed $c = \omega/k$ along the x-axis at the frequency $\omega = 2\pi f$, where f is a frequency in Hz. The double-transformed displacement and stress components $\hat{u}_i(k, y, \omega)$ and $\hat{\sigma}_{ij}(k, y, \omega)$ satisfy a system of ordinary differential equations (ODEs) in y, derived from the elastodynamic equations of motion. Solving of the ODEs subject to the boundary conditions at the top and bottom faces of the plate leads to the displacement and stress components within the plate (e.g. [21,27,26,28]).