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Wave propagation in periodic buckled beams. Part I: Analytical models and numerical simulations

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HIGHLIGHTS

- Periodic buckled beams are dispersive and nonlinear structures.
- Long-wave propagation is described by the Boussinesq and strongly-nonlinear models.
- The solitary wave behavior changes with the support types and buckling level.
- Compressive/tensile supersonic/supersonic solitary waves are predicted.
- Finite element simulations of the structure are used for validation purpose.

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ABSTRACT

Periodic buckled beams possess a geometrically nonlinear, load–deformation relationship and intrinsic length scales such that stable, nonlinear waves are possible. Modeling buckled beams as a chain of masses and nonlinear springs which account for transverse and coupling effects, homogenization of the discretized system leads to the Boussinesq equation. Since the sign of the dispersive and nonlinear terms depends on the level of buckling and support type (guided or pinned), compressive supersonic, rarefaction supersonic, compressive subsonic and rarefaction subsonic solitary waves are predicted, and their existence is validated using finite element simulations of the structure. Large dynamic deformations, which cannot be approximated with a polynomial of degree two, lead to strongly nonlinear equations for which closed-form solutions are proposed.

1. Introduction

Certain buckled structures are periodic media which possess a geometrically (physically) nonlinear, load–deformation relationship and dispersive sources such that solitary waves are possible [1]. They differ from geometrically discrete nonlinear mechanical systems hosting solitary waves as nonlinear penduli [2,3], woodpile periodic structures [4], tensegrity structures [5,6], origami-based metamaterials [7], complex lattices [8–10], and granular media [11,12] in the sense that they are continuous. Buckled structures are also different from continuous systems for which large deformations induce material nonlinearities modeled by the standard continuum theory, incorporating characteristic material lengths [8,13,14] or having reduced dimensions as plates and rods [15–19,3,20].

Periodic buckling is encountered in various systems such as the cooling of thin film fixed to a substrate of different thermal-expansion coefficient [21,22], stretched membranes [23], compressed cylinders [24–26], sandwich panels [27,28], reinforced plates [29], lattices [30–32] or beams resting on elastic foundation [21,33–35]. In the present paper, the emphasis

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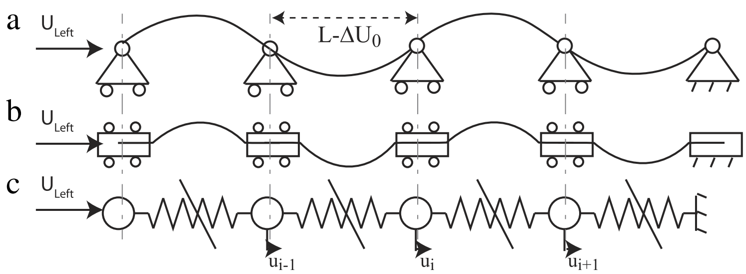

Fig. 1. Guided (only axial displacement allowed) (a) and pinned (both axial displacement and rotation free) (b) support configurations composed of $p = 4$ wavelengths and modeled by a chain of masses and nonlinear springs (c).

is placed on buckling by compression of a slender beam where the elastic foundation is replaced by guided or pinned supports as shown in Fig. 1(a)–(b). Note that in [36,37], wave propagation in undulated beams are also investigated, but the structure are originally curved (no buckling) such that no supports are required for the stability and the medium is linear for small deformations.

In a previous paper [1], and summarized in Section 2.1, we have shown that for a buckled beam described by a chain of masses and nonlinear springs (Fig. 1(c)), only considering dispersion due to periodicity, homogenization of the momentum equation leads to the Boussinesq equation with supersonic solitons as solution. However, although finite element (FE) simulations were in agreement with the supersonic soliton model for a highly-compressed beam, for small and moderate buckling level, a front, characteristic of subsonic nonlinear media, was present; this is in contradiction with our previous model. To this end, we have studied relevant dispersive sources using a semi-analytical dispersion relation [36], a method based on Bloch theorem and a transfer-matrix approach leading to an explicit power series of the wave number and frequency. It is shown that in addition to periodicity, transverse inertia and displacement–rotation coupling are important dispersion sources, and dispersion analyses is summarized in Section 2.2.

The homogenized model and dispersion analyses are summarized in Sections 2.1 and 2.2 respectively.

It is the goal of the present paper to update Boussinesq models including the aforementioned additional dispersive sources, which vary according to pre-compression level and support type, and as it will be shown in Section 2.3, four types of solutions are present, namely compressive supersonic, rarefaction (tensile) supersonic, compressive subsonic and rarefaction subsonic solitary waves.

However, for nonlinear wave propagation with large amplitude or small pre-compression, the Boussinesq equation based on a local approximation of the load–displacement by a second degree polynomial is no longer valid. To this end, Section 2.4 proposed two strongly nonlinear models, one based on a power-law nonlinearity similar to the one used for wave propagation in pre-compressed granular media, and an other one based on the exact load–displacement curve [11]. Existing solution techniques are extended here to account for the additional dispersion sources. Finite-element (FE) models and parameters are given in Section 3 followed in Section 4 by numerical simulations used to validate the derived homogenized models. Discussions and conclusions follow.

This paper, the first part of a series of two, concerns analytical models and numerical simulations for validation purposes and is followed by a second paper focused on experimental validation [38].

2. Solitary waves in buckled structures

The goal of the present paper is to generalize, update, and improve our previous models [1] used to describe wave propagation in one-dimensional (1D) buckled beams (Fig. 1(a)–(b)), and we start with a short review of previous work.

It is assumed for simplicity a slender beam free of shear, with linear-elastic material behavior and constant cross section parameters such that the cross-sectional area is denoted A, the area-moment of inertia I_z, the Young's modulus E, and the density ρ. In order to ensure stability of the p buckled periodic wavelengths, $p + 1$ supports, equally spaced by a distance L are used (Fig. 1(a)–(b)). Although only pinned-supports were considered in [1], the derivation is generalized here to both guided and pinned supports (Fig. 1(a)–(b)) since dispersion depends on the support type [36].

2.1. Solitary waves in buckled structures accounting for axial effects only

Wave propagation is limited to the low-frequency regime treating deformations as quasi-static, and neglecting any phase lags between loads and deformations (see Appendix for an a-posteriori justification). In this regard, wave propagation is described by the axial displacement of the supports, decomposed into $U\{t, x\} = U_0\{x\} + u\{t, x\}$ where $U_0\{x\}$ is the initial displacement due to pre-compression, $u\{t, x\}$ is the finite displacement increment in the buckled configuration, and t and x denote time and space, omitted hereafter for the sake of clarity. The full periodic buckled beam provides a self-similar load–deformation relationship reducing the analysis to a single period. The long wave approximation also justifies the