Wave Motion 66 (2016) 210–219

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Wave propagation in periodic buckled beams. Part II: Experiments

Florian Maurin*, Alessandro Spadoni

Institute of Mechanical Engineering, École Polytechnique Fédérale de Lausanne (EPFL), 1015 Lausanne, Switzerland

HIGHLIGHTS

- Periodic buckled beams are dispersive and nonlinear structures.
- Weakly-nonlinear long-wave propagation is described by the Boussinesq model.
- Experiments are conducted on weakly-buckled, guided-supported beams.
- Experimental results are used to validate analytical and numerical models.

ARTICLE INFO

Article history:
Received 2 October 2015
Received in revised form 17 April 2016
Accepted 19 May 2016
Available online 30 May 2016

Keywords:
Buckled periodic beam
Experiment
Solitary wave

ABSTRACT

Buckled periodic beams possess a geometrically nonlinear, load–deformation relation and intrinsic length scales such that stable, nonlinear waves are possible. In part I of this paper, a model has been derived that predict compressive/rarefaction, supersonic/supersonic solitary waves, varying the level of compression and the support type (guided or pinned). Although this work has been validated by simulating the structure with finite-elements, in the present paper, investigations are done experimentally, focusing on the guided-supported, slightly-buckled beam.

1. Introduction

Periodic buckled beams (Fig. 1) are dispersive and geometrically (physically) nonlinear structures that admit solitary waves [1,2]. In [1], modeling the buckled beam by a chain of masses and nonlinear springs taking into account transverse inertial and coupling effects, it is shown that homogenization of the discretized system leads to the doubly dispersive Boussinesq equation. This equation admits compressive supersonic, rarefaction (tensile) supersonic, compressive subsonic and rarefaction subsonic solitary waves, and these four waves are all possible in the same structure, the buckled beam, simply by varying the pre-compression level and support type (guided or pinned, see Fig. 1). While finite-element (FE) simulations of the structure have been performed validating the derived models, the aim of the proposed paper is to repeat this work experimentally.

Solitary mechanical waves have been observed experimentally in discrete media as nonlinear penduli [3,4], woodpiles [5], complex lattices [6], and granular media [7,8], and the dispersion arise from the periodicity. However, for buckled beams, the beam thickness and the curvature are also important dispersive sources and have to be taken into account as well.

From the static solution of beam theory, a straight beam compressed by an axial load admits analytically an infinite number of periodic buckling modes [9]. However, due to imperfections (e.g. geometry not perfectly straight), only

* Corresponding author.
E-mail address: florian.maurin@gmail.com (F. Maurin).

http://dx.doi.org/10.1016/j.wavemoti.2016.05.009

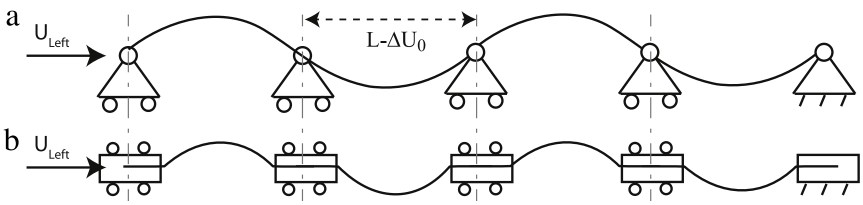

Fig. 1. Guided (only axial displacement allowed) (a) and pinned (both axial displacement and rotation free) (b) support configurations composed of $p = 4$ wavelengths.

the first buckling mode is stable, but is not periodic. Whereas stability of high-order modes can be ensured via an elastic foundation [10–13], cooling of thin film fixed to a substrate of different thermal-expansion coefficient [10,14], or compressing a beam between two fixed walls [15], these additional media bring complex interactions as friction or secondary paths for wave propagation. In the literature, two/three dimensional (2D/3D) self-supported periodic buckled structures are also reported experimentally, such as stretched membranes [16], twisted and stretched strips [17,18], compressed cylinders [19,20], sandwich panels [21,22], reinforced plates [23] and lattices [24,25].

Here, to ensure the stability of the initially straight beam, the use of additional supports that prevent transverse displacements is proposed, as shown in Fig. 1. While transverse displacements of the supports are forbidden and axial ones are free, no constrain applies on the rotation such that either pinned or guided supports can be used (Fig. 1), and both configurations are investigated in [1,26]. Although the support type changes the speed behavior (supersonic or subsonic medium) and the solitary wave type (compressive or rarefaction) [1], in the present paper, only guided supports that are easier to realize experimentally are investigated.

From material considerations, an experimental limitation is the maximum reachable buckling level. While in [1] it is shown that wave characteristics are also dependent on pre-compression, experimentally, only small buckling levels can be tested for metallic structures. Indeed, when the buckling is too high such that the material reaches its yield limit, plastic hinges occur and the structure collapses [27]. Hyperelastic materials can be used to avoid plasticity [28], but are not investigated here.

This article is organized as follows: while the generalized Boussinesq model derived in [1] is shortly reviewed in Section 2, the experimental setup is shown in Section 3. Results are presented in Section 4, investigating first static buckling and focusing then on the wave profile and its characteristics. Discussion and conclusion follow.

2. Analytical models

In part I of this paper [1], analytical models describing wave propagation in buckled beams have been derived and are shortly reviewed here. Modeling the buckled beam by a chain of masses and nonlinear springs, and accounting for transverse inertial effects [1,26], homogenization of the momentum equation leads in the case of guided supports to the Boussinesq equation [1]:

$$\xi_{tt} = C_0^2 \xi_{xx} + 2C_0 \gamma \xi_{xxxx} + \sigma(\xi \xi_x)_x, \tag{1}$$

where $C_0^2 = \frac{P'\{\Delta U_0\} L_0^2}{m}$ is the linear speed, $\sigma = \frac{P''\{\Delta U_0\} L_0^3}{m}$ the coefficient of nonlinearity, and $\gamma = -\frac{a_4 L_0^2 C_0}{2}$ the coefficient of dispersion. a_4 is the dimensionless dispersion coefficient that accounts for all the dispersion sources, and is determined numerically from the semi-analytical dispersion relation [1,26]. At the static level, $L_0 = L - \Delta U_0$ is the length between two consecutive supports after applying pre-compression, $P\{\Delta U_0\}$ represents the load–displacement relation with ΔU_0 the relative displacement between two consecutive supports and $\chi_0 = \Delta U_0 / L$ is the initial strain level. At the dynamic level, $\xi = -U_x = -\Delta U / L_0 = -u_x + \chi_0$ is the compressive strain, and U are the total support displacements. Under the long-wave assumption, Eq. (1) admits the solitary-wave solution [1]:

$$\Delta \xi = \Delta \xi_m \operatorname{sech}^2 \left\{ \Lambda^{-1}(x - Vt) \right\}, \tag{2}$$

where $\Delta \xi = \xi - \chi_0$, $\Delta \xi_m = \xi_m - \chi_0$, and ξ_m is the maximum strain. The solitary wave phase speed V is:

$$V = \sqrt{C_0^2 + \sigma \Delta \xi_m / 3}, \tag{3}$$

and its characteristic width Λ is

$$\Lambda = \sqrt{24 C_0 \gamma / (\sigma \Delta \xi_m)}. \tag{4}$$

While the Boussinesq model assumes the load–displacement relation ($P\{\Delta U\}$) described by a second-degree polynomial, it is shown in [1] that this model is not accurate for high-amplitude waves and alternative methods using the potential of $P\{\Delta U\}$ described by either a power law or an arbitrary function are proposed. However, since these additional models are more complex to implement and interpret, experimental parameters are restricted here where the Boussinesq model applies.