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The mathematical model of the radial artery blood pressure waveform through monitoring of the age-related changes

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HIGHLIGHTS

- A mathematical model of blood pressure for different age groups is proposed.
- Pressure waveform is a superposition of forward and backward waves.
- The model shows age-related changes in blood pressure waveform.
- Arterial changes may cause an increase in pulse wave reflection.

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ABSTRACT

Age-related changes in blood vessels affect the pulse wave propagation. These changes may cause an increase in wave reflection which leads to amplification of pulse pressure. The pulse pressure changes are associated with certain vascular diseases. Here we present a mathematical model of blood pressure for different age groups. The model is based on one-dimensional wave propagation theory and assumes that the pressure waveform is a superposition of forward propagating wave and backward waves from many reflection sites. The model is based on experimental data obtained by direct measurement of radial artery blood flow. The model clearly shows age-related changes in blood pressure waveform. The results of mathematical model correlate with the radial pressure waveform data. The model can be used in cases where it is not possible to measure the pressure due to movement of subject. Application of model to the direct blood flow measurements data allows the real-time pressure waveform monitoring. Furthermore, this approach enables monitoring of changes in pressure waveform due to the effects of medications.

1. Introduction

The physics of fluid flow has been investigated over the past two centuries [1,2]. One of the important fields of research deals with the propagation of pulse waves through the fluid-fill elastic tube. The motivation for this research is the proper biophysical understanding of arterial hemodynamics.

Previous studies have shown that pulsatile pressure and flow waveforms contain important information about the heart, the vascular system and a certain cardiovascular diseases [3–6]. Many changes that occur in pressure and flow waveforms

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are associated with the mechanical properties of blood vessels and can lead to answers about the age-related changes in the cardiovascular system or changes in cardiovascular dynamics [5–7]. As the systemic arterial pressure increases with age, structure and properties of arteries undergo some characteristic changes. For example, changes in elastin and collagen lead to a reduction of elastic properties of arteries [6,8,9]. Increase in wave reflection on the large arteries branching is one of the direct indicators of reduced arterial elasticity. Wave reflection is associated with pulse pressure amplification and can be detected in the pulse waveform in hypertension and atherosclerosis [3,10,11].

There are various mathematical models, and each provides answers to specific questions of blood flow. The simplest models that include Windkessel, so-called lumped models [12–14], can be used to explain the whole arterial dynamics of flow [15,16], but cannot be useful for investigation of wave reflections and travel phenomenon. To describe blood flow and blood pressure in a specific arterial space, nonlinear one-dimensional (1D) [17–20] and three dimensional (3D) [21,22] models are used.

Predicting and understanding changes in pressure in the arteries have a great role in medical diagnostics and research that tries to explain causes of cardiovascular diseases. There are several sites on the body where the blood pressure waveform can be recorded. The most commonly used sites are above carotid artery in the neck and above the radial artery in the wrist. In these areas, bone and ligaments support the artery, and the artery does not move when external pressure is applied.

Continuous non-invasive blood pressure measurements are still not possible without arterial compression (arterial tonometer). Further, the respondent must be in a stationary position during the measurement. On the other hand, non-invasive blood flow measurement is very simple to perform by using optical properties of blood with highly reproducible results. Using a recorded blood flow signal, various mathematical and physical models simulate a shape of pressure waveform in artery [16,23–25]. The advantages of blood flow measurements are the possibility to perform long-term measurements and the respondent can be in any position, even in motion. Therefore, the development of mathematical and physical models of the pressure is of great importance for cardiovascular research.

In this work we present the results of pulse pressure waveform model constructed according to the data recorded by direct measurement of blood flow at the radial artery. The model is based on the transmission line theory and on the assumption that the pressure waveform is a superposition of the forward propagating wave and the backward waves from several reflection sites [26]. The results of the model show clear differences in pulse pressure waveform shapes for the three groups of examined subjects: young, middle-aged and elderly. In addition, we demonstrate here the identification and separation of the reflected wave in radial artery flow signal arising from the arterial branching.

The proposed mathematical model is very promising as a diagnostic tool. Using computer technology it is possible to simulate the physiological signal waveforms that are very difficult to measure with noninvasive methods. Presented mathematical model may be applied in monitoring of age-related changes in the cardiovascular system. This model can be used as a long-term monitoring of effects of drugs, in cardiovascular therapy. Also, in the elderly and immobile patients, the model can be very useful to the physician.

2. Methods

2.1. Wave propagation in a single artery

The behavior of blood in a short segment of human artery can be simulated as a one-dimension, a Newtonian, incompressible fluid flow in elastic tube. Starting from continuity equation

$$\frac{\partial Q}{\partial z} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

and linearized momentum equation [27]

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \mu \cdot \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} \right) \tag{2}$$

with assumptions that the radial and tangential motion of fluid are neglected and the pressure gradient having a form

$$-\frac{\partial p}{\partial z} = p_0 \cdot e^{i\omega t} \tag{3}$$

where A is a cross-sectional area, Q is a flow (volume/time) through the elastic tube, p_0 is a complex constant denoting an amplitude of the pressure gradient and ω is an angular frequency.

The solution given by Womersley [27] is,

$$u = u(r, z, t) = \frac{A}{i\omega\rho} \left[1 - \frac{J_0\left(i^{\frac{3}{2}}\alpha y\right)}{J_0\left(i^{\frac{3}{2}}\alpha\right)} \right] e^{i\omega t} \tag{4}$$

where $\alpha^2 = \frac{r_0^2 \omega \rho}{\mu}$ (α—Womersley number), $y = \frac{r}{r_0}$, r_0 is the internal arterial radius, J_0, J_1 are the Bessel functions of order zero and first of the first kind and μ is the blood viscosity.