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Generation of acoustic solitary waves in a lattice of Helmholtz resonators

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HIGHLIGHTS

- Numerical modeling of nonlinear hyperbolic systems with nonlinear relaxation.
- Experimental investigation of nonlinear propagation.
- Validation of a theoretical model involving acoustic solitary waves.

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ABSTRACT

This paper addresses the propagation of high amplitude acoustic pulses through a 1D lattice of Helmholtz resonators connected to a waveguide. Based on the model proposed by Sugimoto (1992), a new numerical method is developed to take into account both the nonlinear wave propagation and the different mechanisms of dissipation: the volume attenuation, the linear viscothermal losses at the walls, and the nonlinear absorption due to the acoustic jet formation in the resonator necks. Good agreement between numerical and experimental results is obtained, highlighting the crucial role of the nonlinear losses. Different kinds of solitary waves are observed experimentally with characteristics depending on the dispersion properties of the lattice.

1. Introduction

The dynamics of nonlinear waves in lattices has been the object of a great interest in the scientific community. This theme has stimulated researches in a wide range of areas, including the theory of solitons and the dynamics of discrete networks. Works have been led in electromagnetism and optics [1], and numerous physical phenomena have been highlighted, such as dynamical multistability [2–4], chaotic phenomena [5,6], discrete breathers [7–9] and solitons or solitary waves [10,11]; for a review, see [12]. Solitary waves have been observed and studied first for surface wave in shallow water [13]. These waves can propagate without change of shape and with a velocity depending of their amplitude [14]. This phenomenon has been studied in many physical systems, for instance in fluid dynamics, optics, plasma physics. For a review, see [15] and the citations in [16].

In the field of acoustics, numerous works have shown the existence of solitary waves in uniform or inhomogeneous rods [17–19], periodic chains of elastics beads [20–24], periodic structures such as lattices or crystals [25–27], elastic

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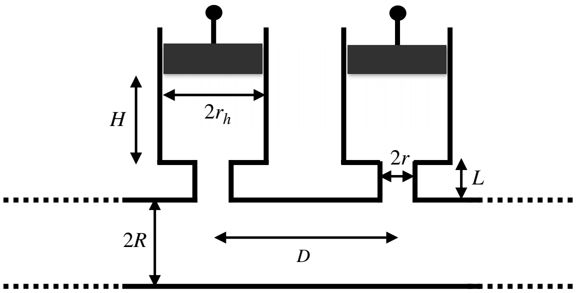

Fig. 1. Sketch of the guide connected with Helmholtz resonators.

layers [28–30], layered structures coated by film of soft material [31] and microstructured solids [32]. As we can see, most studies concern elastic waves in solids. Indeed, only a few works deal with acoustic waves in fluid, even if experimental observations of solitary waves have been made in the atmosphere [33–35] or in the ocean [36–38].

One reason of this lack originates from the fact that the intrinsic dispersion of acoustic equations is too low to compete with the nonlinear effects, preventing from the occurrence of solitons. To observe the latter waves, geometrical dispersion must be introduced. It has been the object of the works of Sugimoto and his co-authors [39–42], where the propagation of nonlinear waves was considered in a tube connected to an array of Helmholtz resonators. A model incorporating both the nonlinear wave propagation in the tube and the nonlinear oscillations in the resonators has been proposed. Theoretical and experimental investigations have shown the existence of acoustic solitary waves [39].

The present study extends the work of Sugimoto. We examine the validity of his theoretical model to describe the propagation of nonlinear acoustic waves in a tunnel with Helmholtz resonators. For this purpose, we develop both a new numerical method and real experiments. Compared with our original methodology presented in [43], improvements are introduced to model numerically the attenuation mechanisms. The combination of highly-accurate numerical simulations and experimental results enables to study quantitatively the generation of solitary waves, and also to determine the role of the different physical phenomena (such as the linear and nonlinear losses) on wave properties.

The paper is organized as follows. Section 2 introduces the model of Sugimoto [42]. Section 3 presents the evolution equations. The nonlocal fractional derivatives modeling the viscothermal losses are transformed into a set of memory variables satisfying local-in-time ordinary differential equations. Sugimoto's model is then transformed into a first-order system of partial differential equations. Section 4 details the numerical methods. The coefficients of the memory variables are issued from a new optimization procedure, which ensures the decrease of energy. A splitting strategy is then followed to integrate the evolution equations. Compared with [43], another novelty concerns the integration of a nonlinear differential equation describing the nonlinear losses. Section 5 introduces the experimental setup, the acquisition chain, and some validation tests. Section 6 compares the experimental results and the simulated results, confirming the validity of the theoretical model [40] and the existence of acoustic solitary waves.

2. Problem statement

2.1. Configuration

The configuration under study is made up of an air-filled tube connected with uniformly distributed cylindrical Helmholtz resonators (Fig. 1). The geometrical parameters are the radius of the guide R; the axial spacing between resonators D; the radius of the neck r; the length of the neck L; the radius of the cavity r_h; and the height of the cavity H. The cross-sectional area of the guide is $A = \pi R^2$ and that of the neck is $B = \pi r^2$, the volume of each resonator is $V = \pi r_h^2 H$. Corrected lengths are introduced: $L' = L + 2r$ accounts for the viscous end corrections, and the corrected length $L_e = L + \eta$ accounts for the end corrections at both ends of the neck, where $\eta \approx 0.82\,r$ is determined experimentally [40]. The reduced radius is:

$$R^* = \frac{R}{1 - \frac{R}{2D}\frac{B}{A}} = \frac{R}{1 - \frac{r^2}{2DR}}. \tag{1}$$

The physical parameters are the ratio of specific heats at constant pressure and volume γ; the pressure at equilibrium p_0; the density at equilibrium ρ_0; the Prandtl number Pr; the kinematic viscosity ν; and the ratio of shear and bulk viscosities μ_v/μ. The linear sound speed a_0, the sound diffusivity ν_d, the dissipation in the boundary layer C, and the characteristic angular frequencies of the resonator ω_0 and ω_e, are given by:

$$\begin{aligned} &a_0 = \sqrt{\frac{\gamma\, p_0}{\rho_0}}, \qquad \nu_d = \nu\left(\frac{4}{3} + \frac{\mu_v}{\mu} + \frac{\gamma - 1}{\mathrm{Pr}}\right), \qquad C = 1 + \frac{\gamma - 1}{\sqrt{\mathrm{Pr}}}, \\ &\omega_0 = a_0\sqrt{\frac{B}{LV}} = a_0\,\frac{r}{r_h}\,\frac{1}{\sqrt{LH}}, \qquad \omega_e = \sqrt{\frac{L}{L_e}}\,\omega_0. \end{aligned} \tag{2}$$